03 Concurrent Objects

CS 6868: Concurrent Programming

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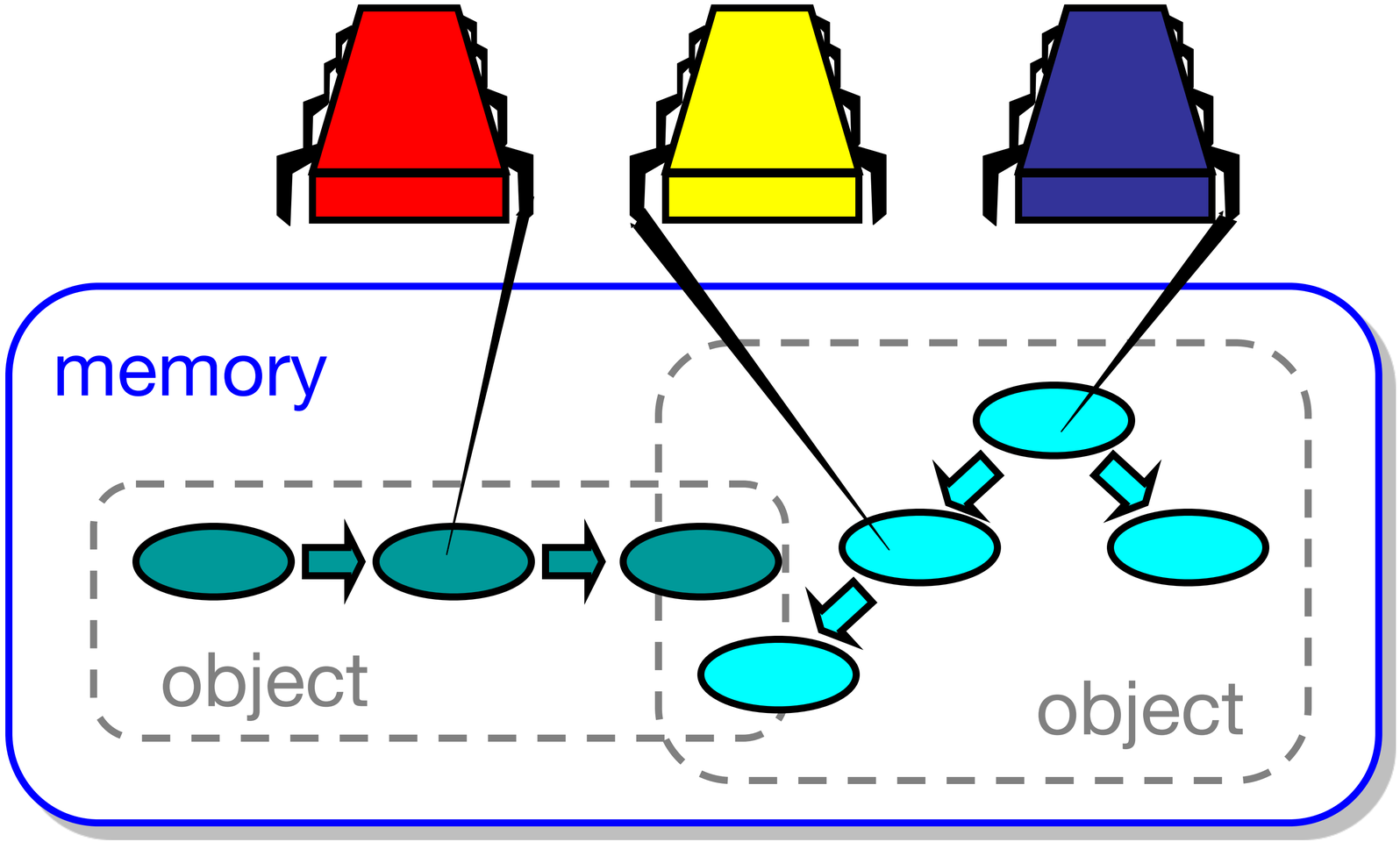
Concurrent Computation



Objectivism

- What is a concurrent object?
  - How do we ***describe*** one?
  - How do we ***implement*** one?
  - How do we ***tell if we're right***?

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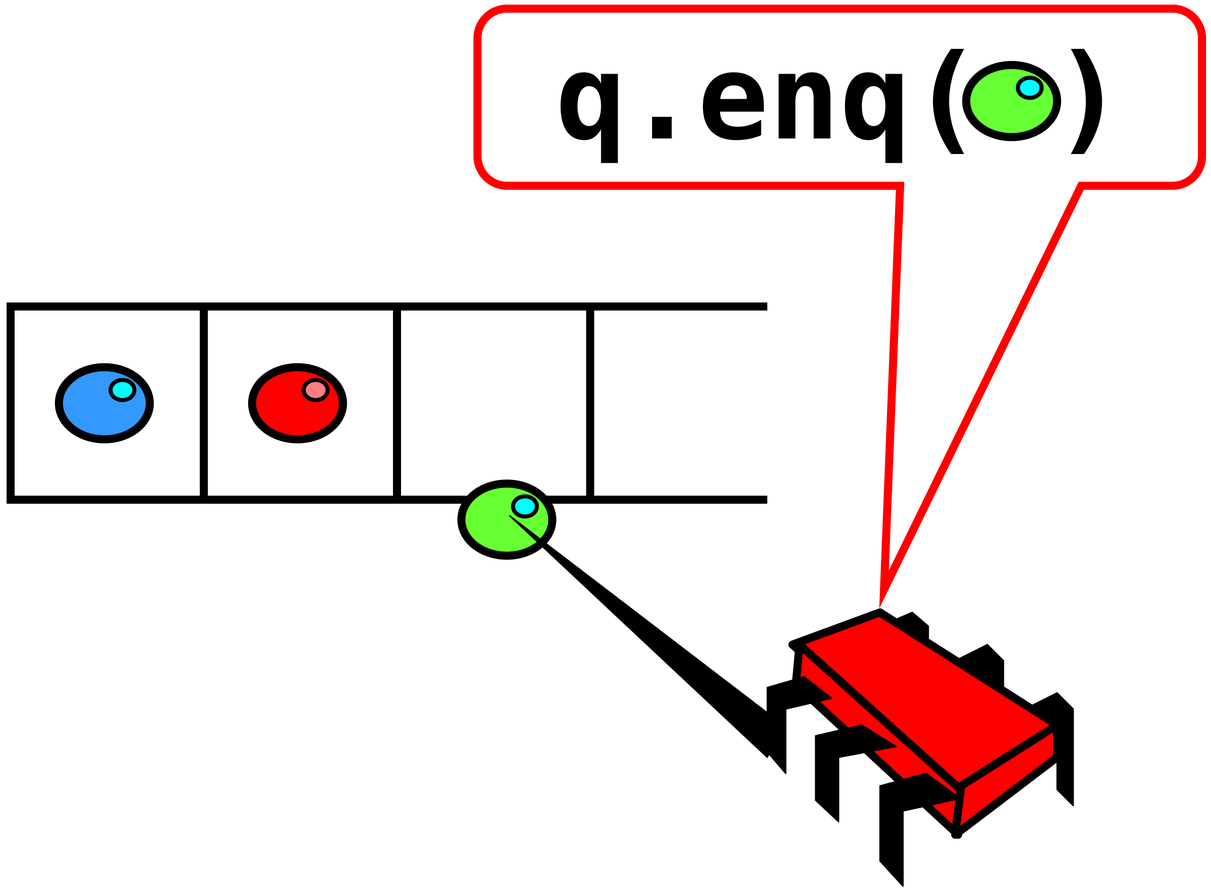
Concurrent Queues

FIFO Queue — Enqeue method



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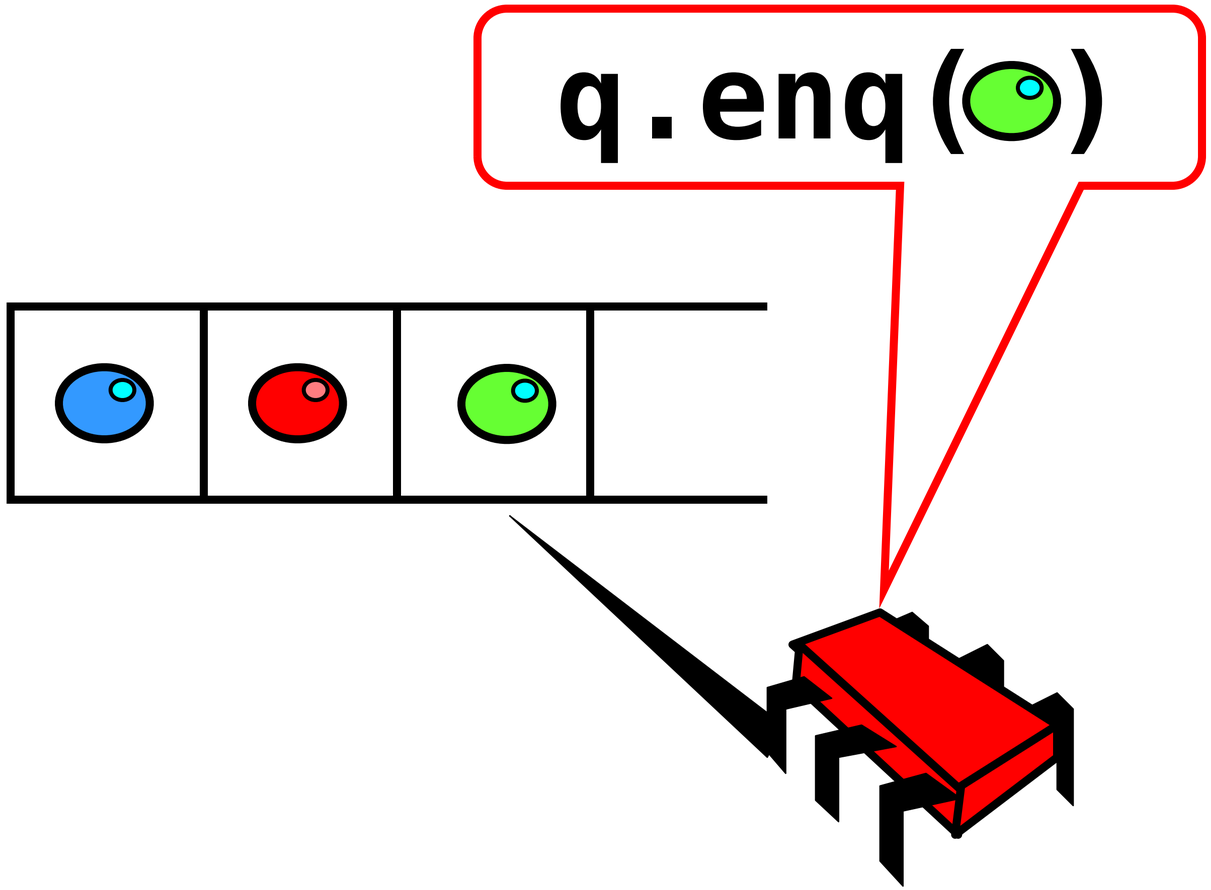
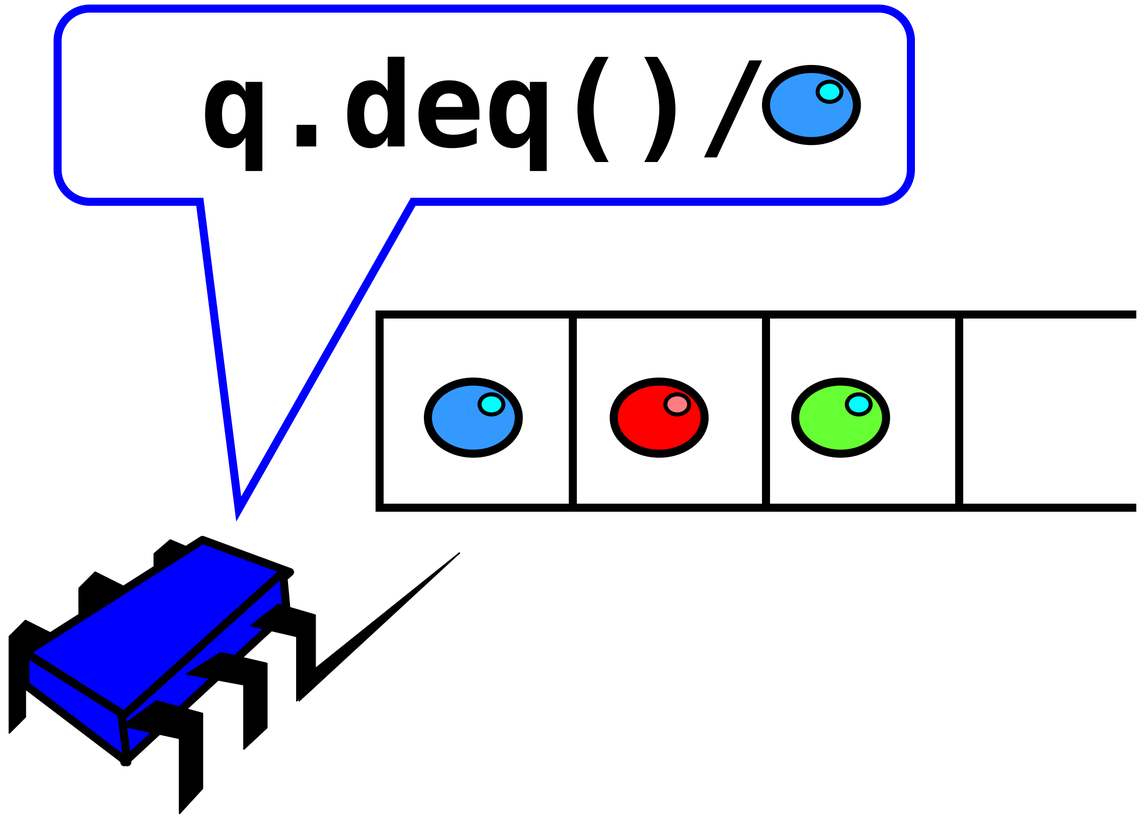


FIFO Queue — Dequeue method



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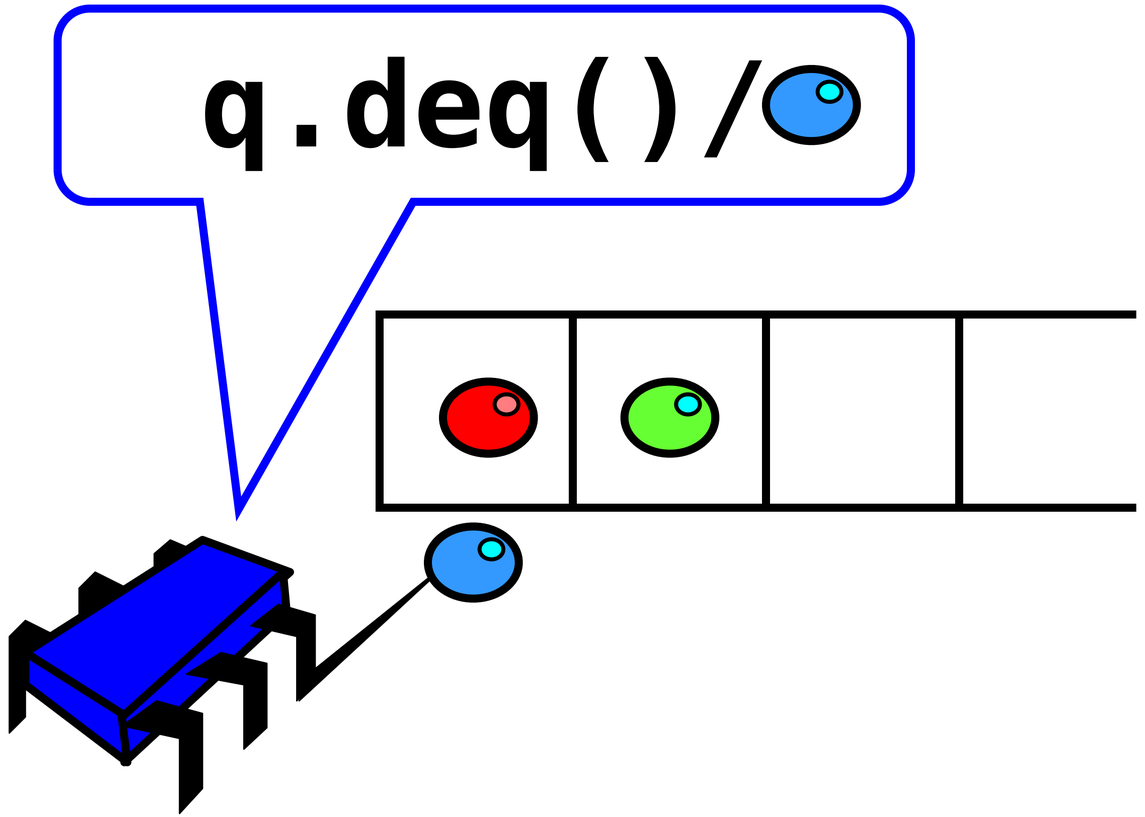
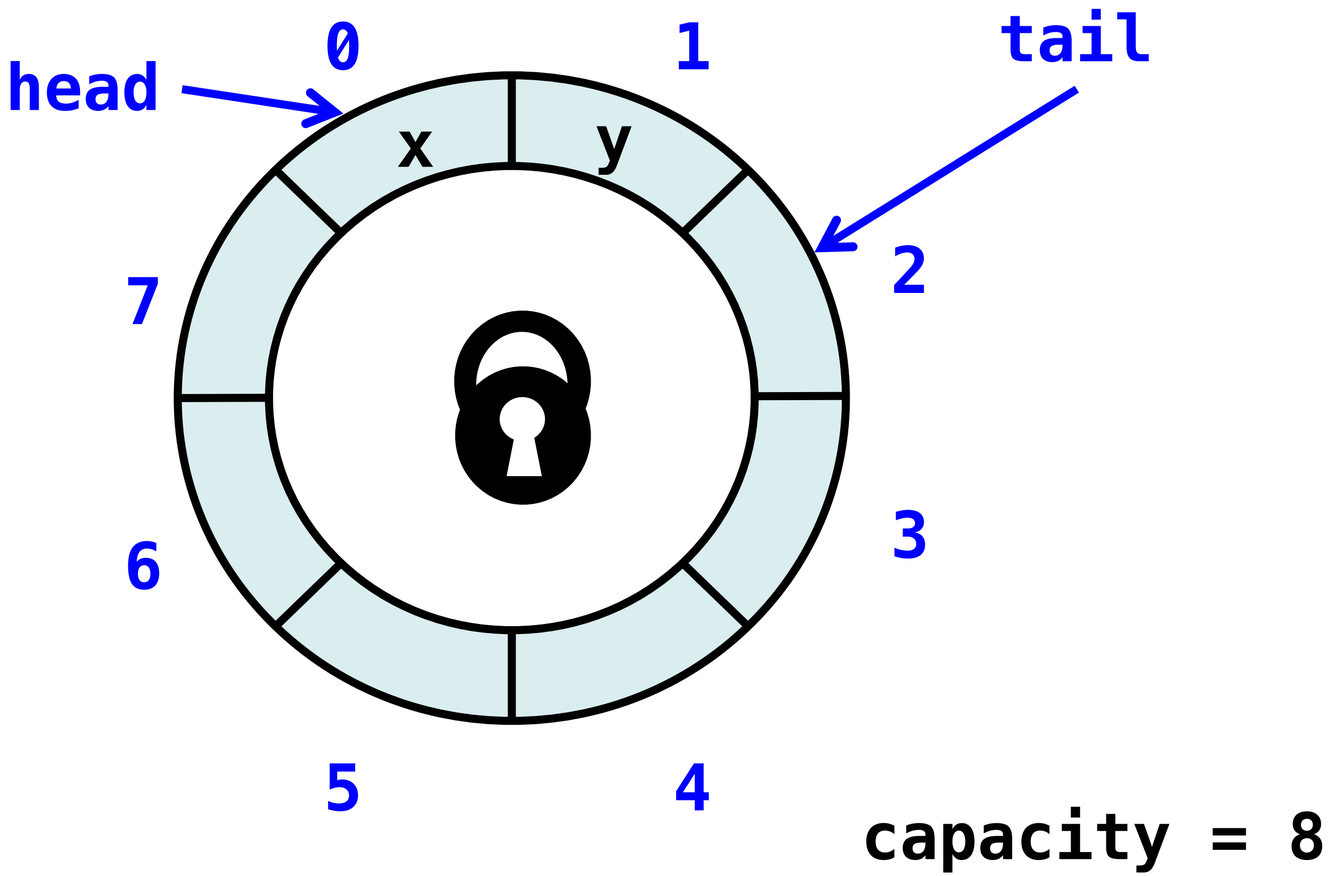


Lock-based Queue



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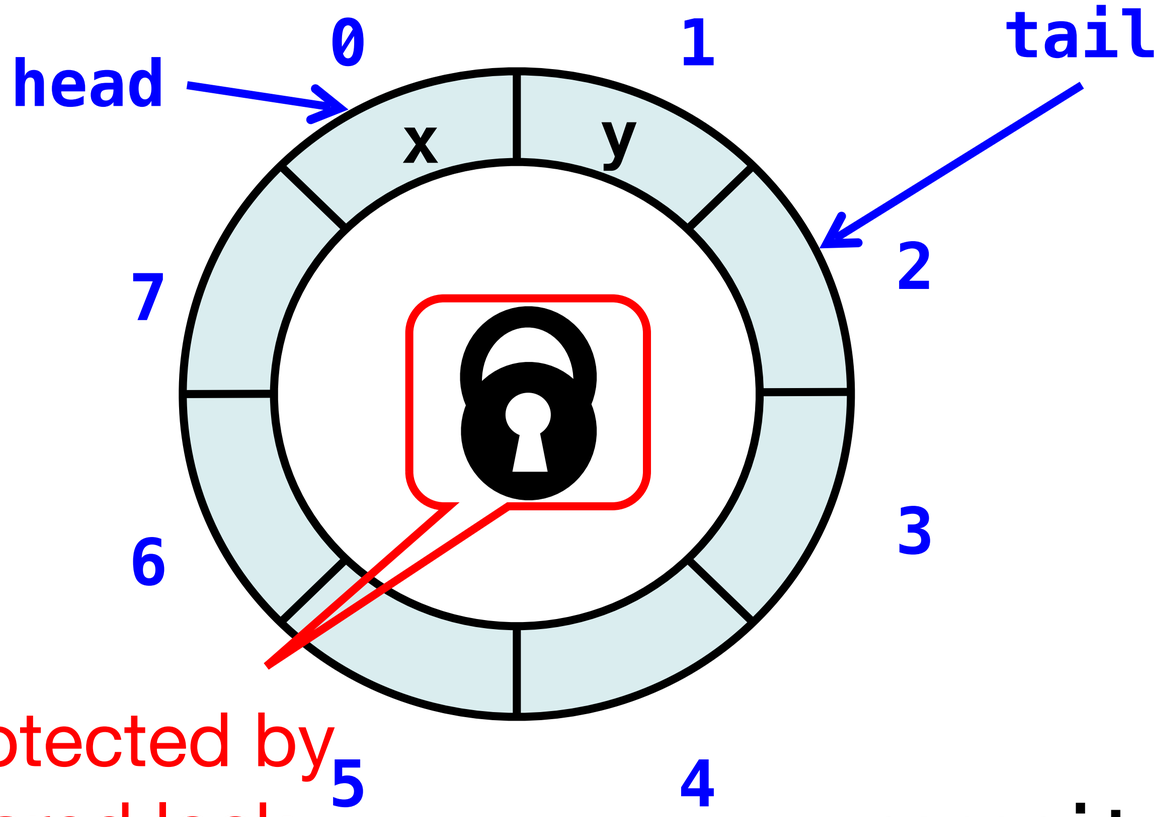


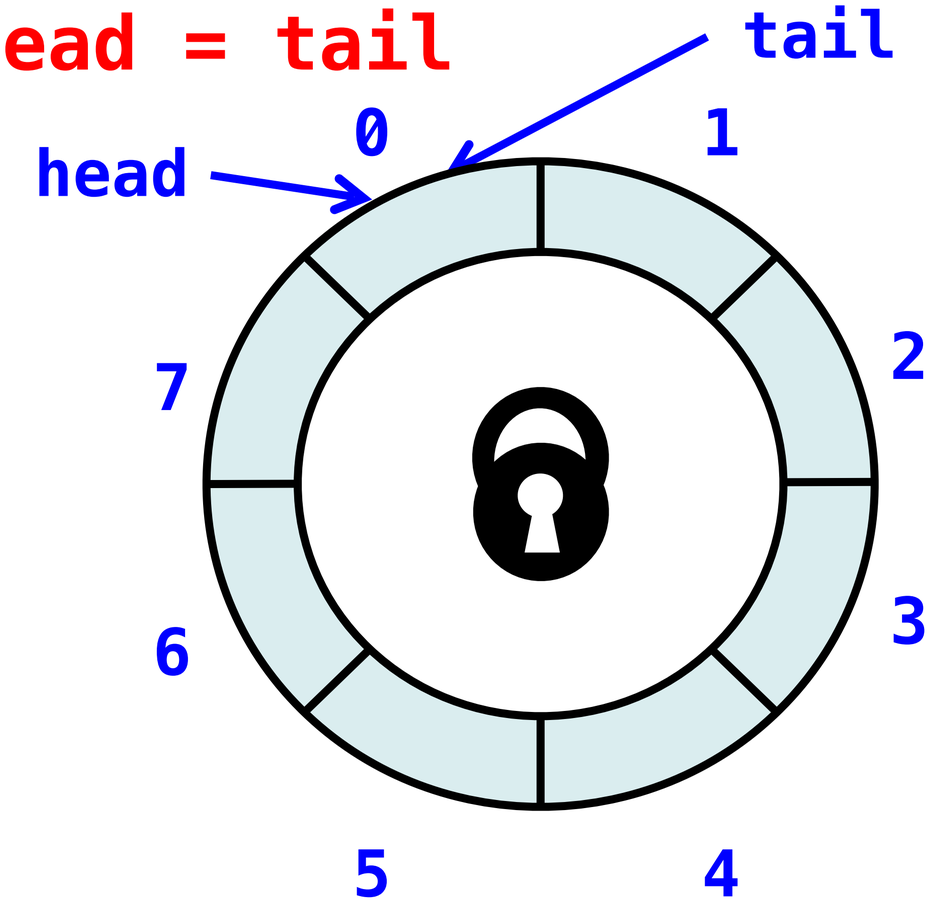
Fields protected by
single shared lock

capacity = 8

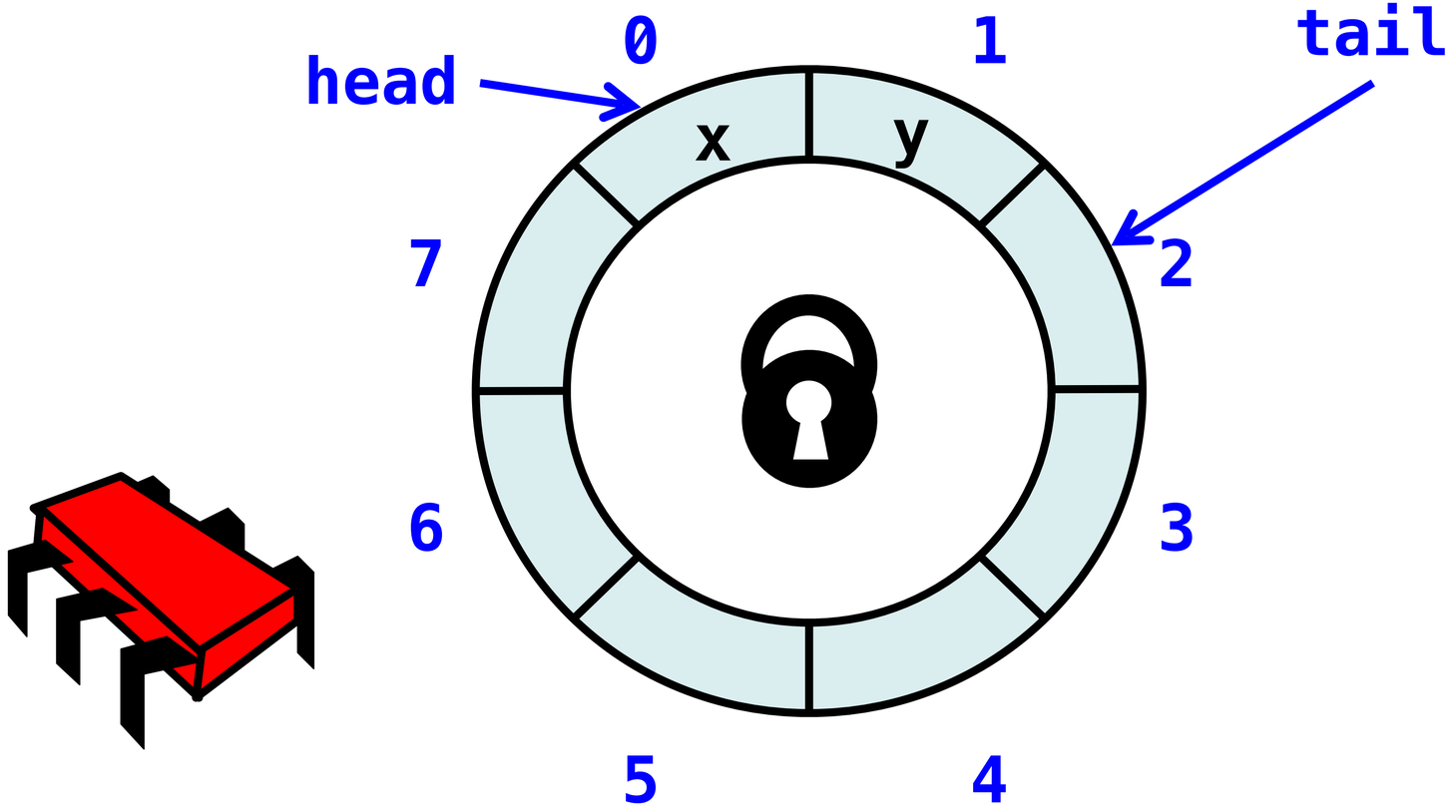
Lock-based Queue

```
exception Full
exception Empty

type 'a t = {
  items : 'a option array;
  capacity : int;
  mutable head : int;
  mutable tail : int;
  lock : Mutex.t;
}

let create capacity =
  {
    items = Array.make capacity None;
    capacity;
    head = 0;
    tail = 0;
    lock = Mutex.create ();
  }
```

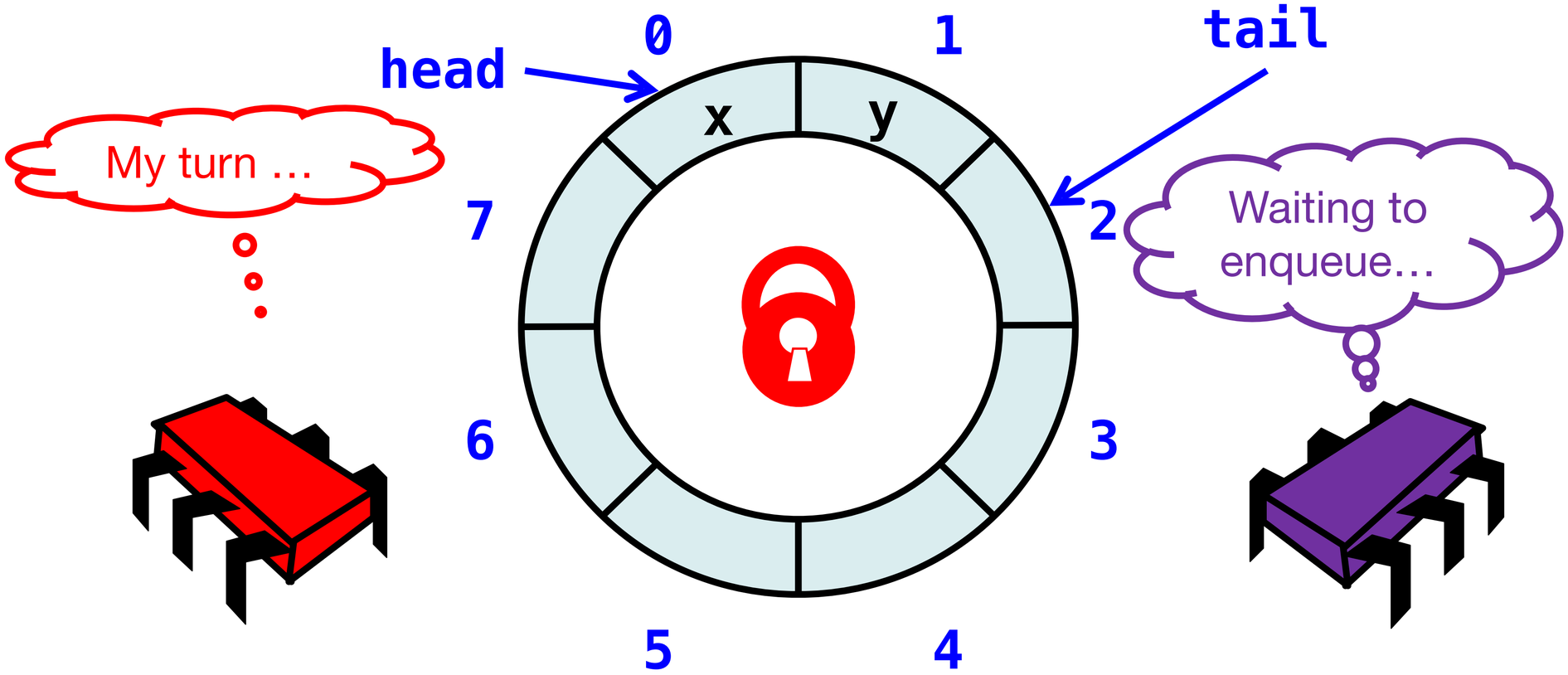
Initially: **head = tail**



Lock-based Queue — deq() operation



Acquire Lock

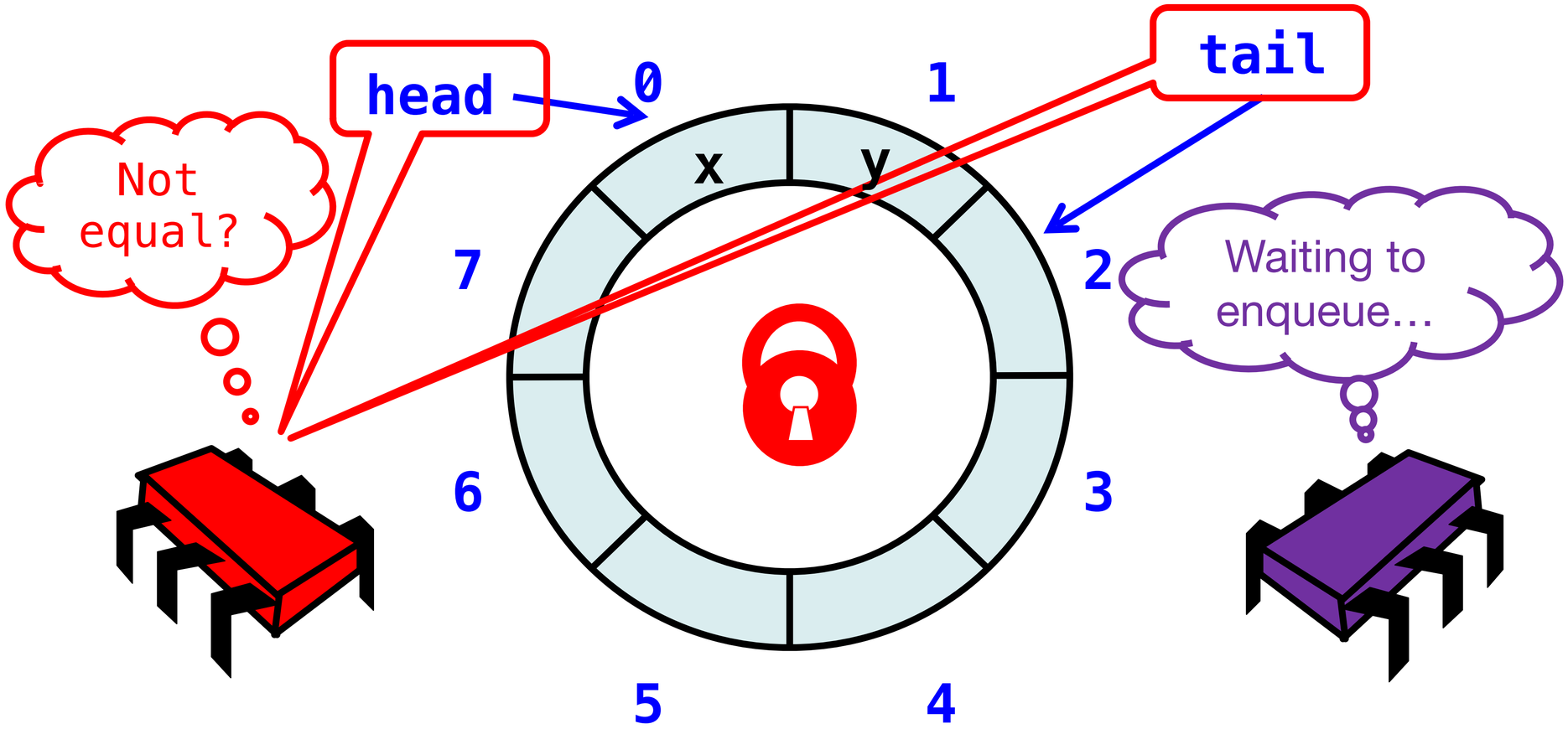


Acquire Lock

```
let deq q =
  Mutex.lock q.lock;
  try
    if q.tail = q.head then
      raise Empty;
    match q.items.(q.head mod q.capacity) with
    | None -> assert false
    | Some x ->
        q.head <- q.head + 1;
        Mutex.unlock q.lock;
        x
  with e ->
    Mutex.unlock q.lock;
    raise e
```

Acquire lock at method start

Check if non empty



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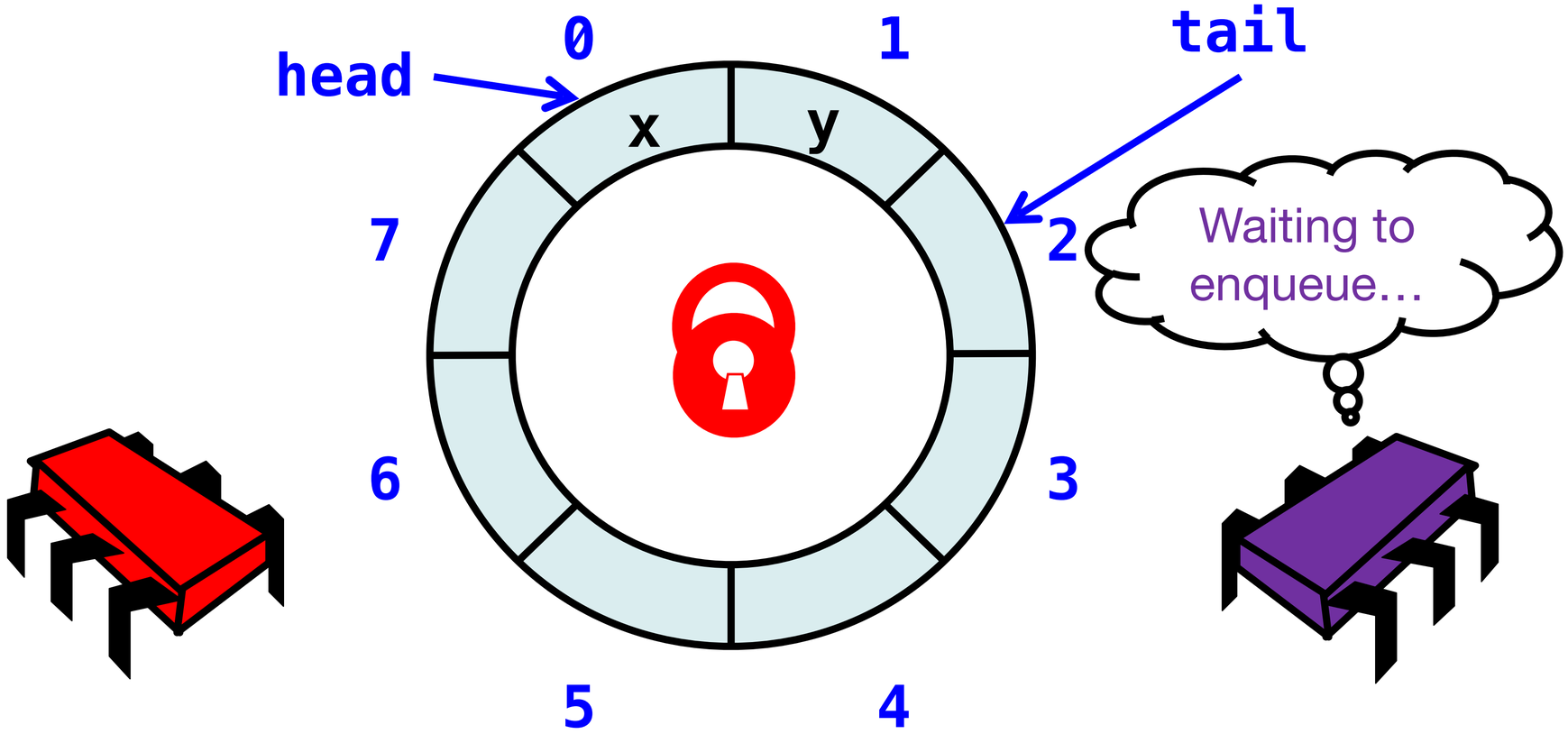
```

If queue empty throw exception

In case of exceptions, lock released here

Modify the queue



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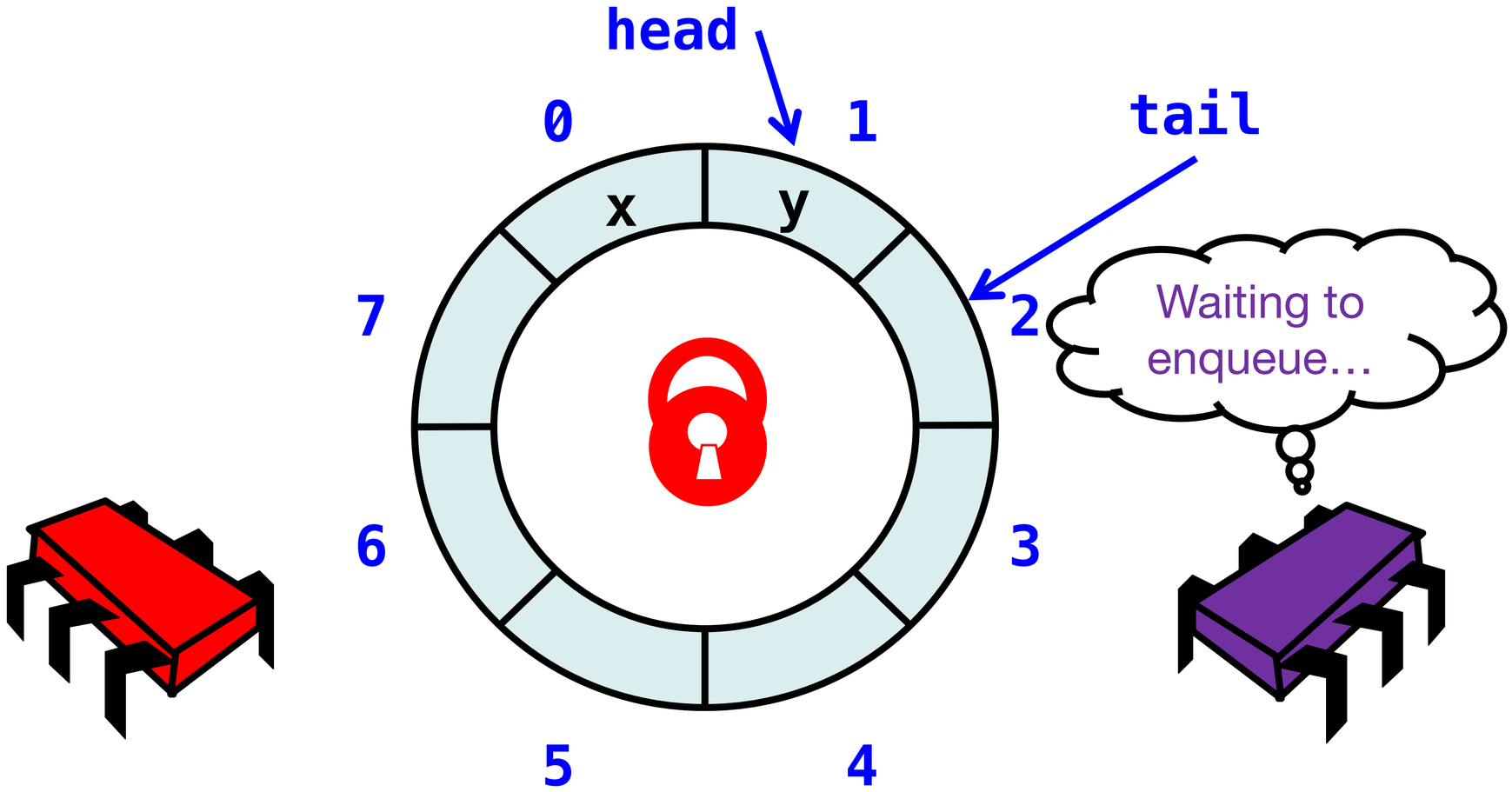

Modify the queue

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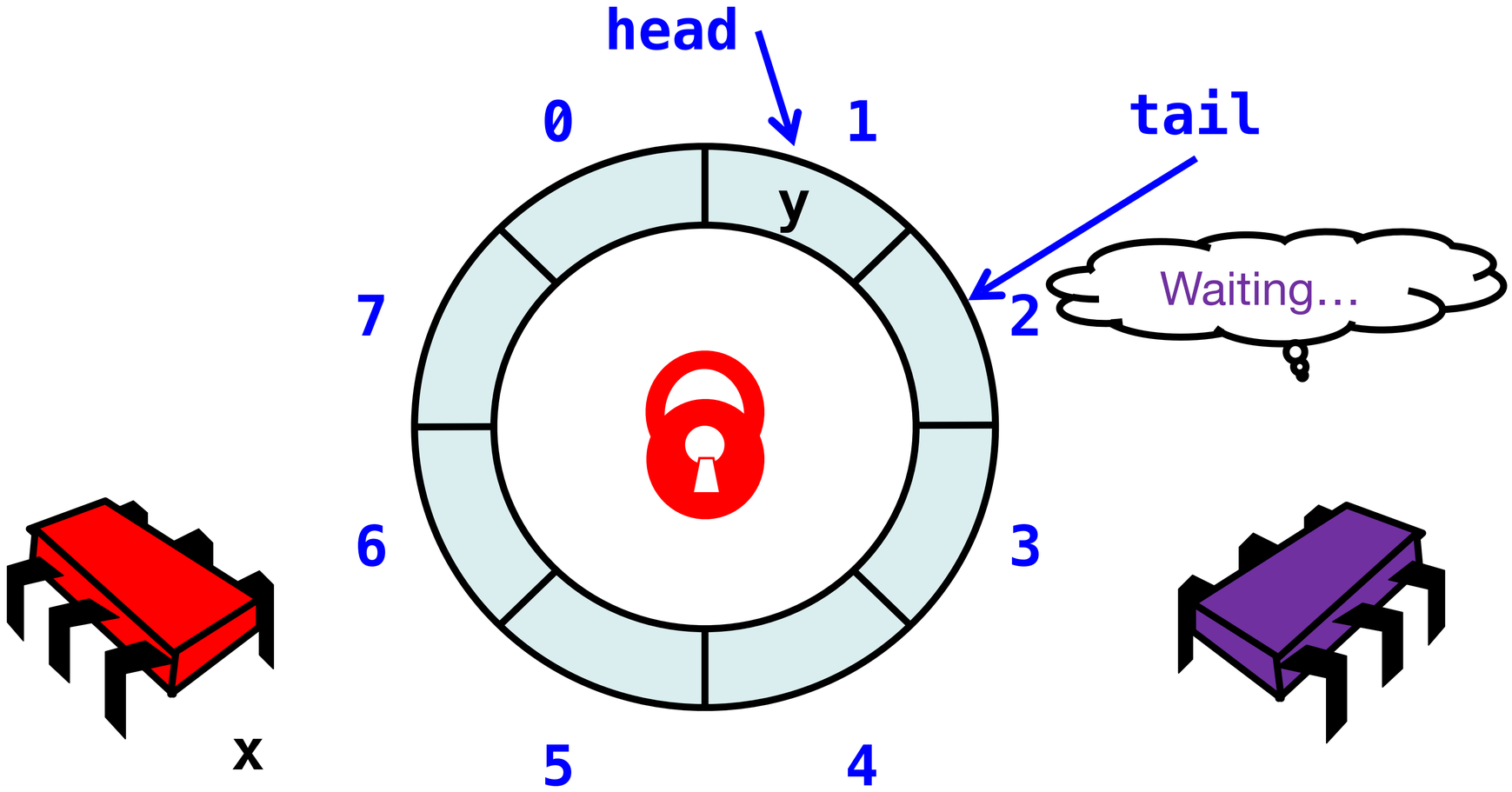
Queue not empty?

Remove item “x” and update head

Release the lock and return item



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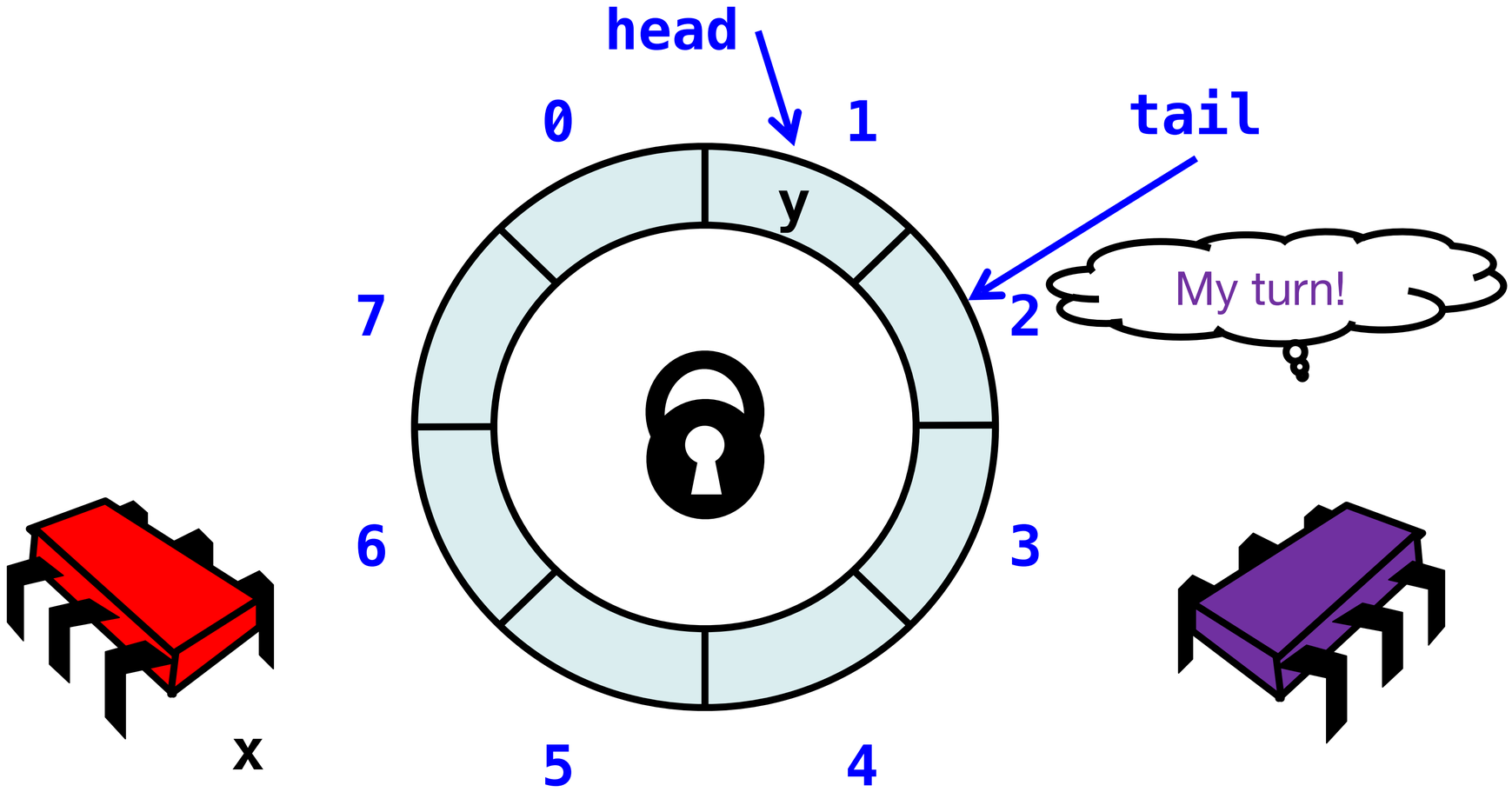

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Unlock and return item “x”

Implementation — deq()

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Should be correct because modifications are mutually exclusive…

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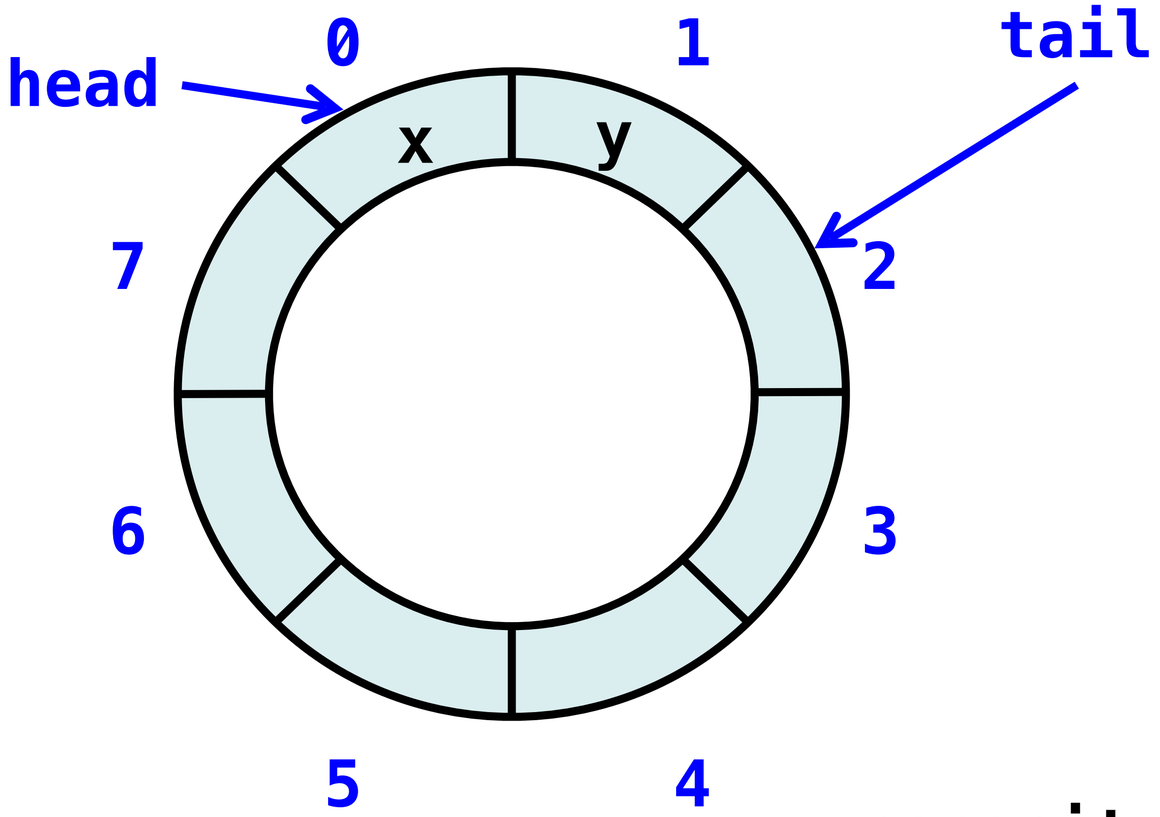
Demo

Consider the following implementation

- The same thing without mutual exclusion
- For simplicity, only ***two*** threads
  - One thread enq only
  - The other deq only

Wait-free 2-thread queue



capacity = 8

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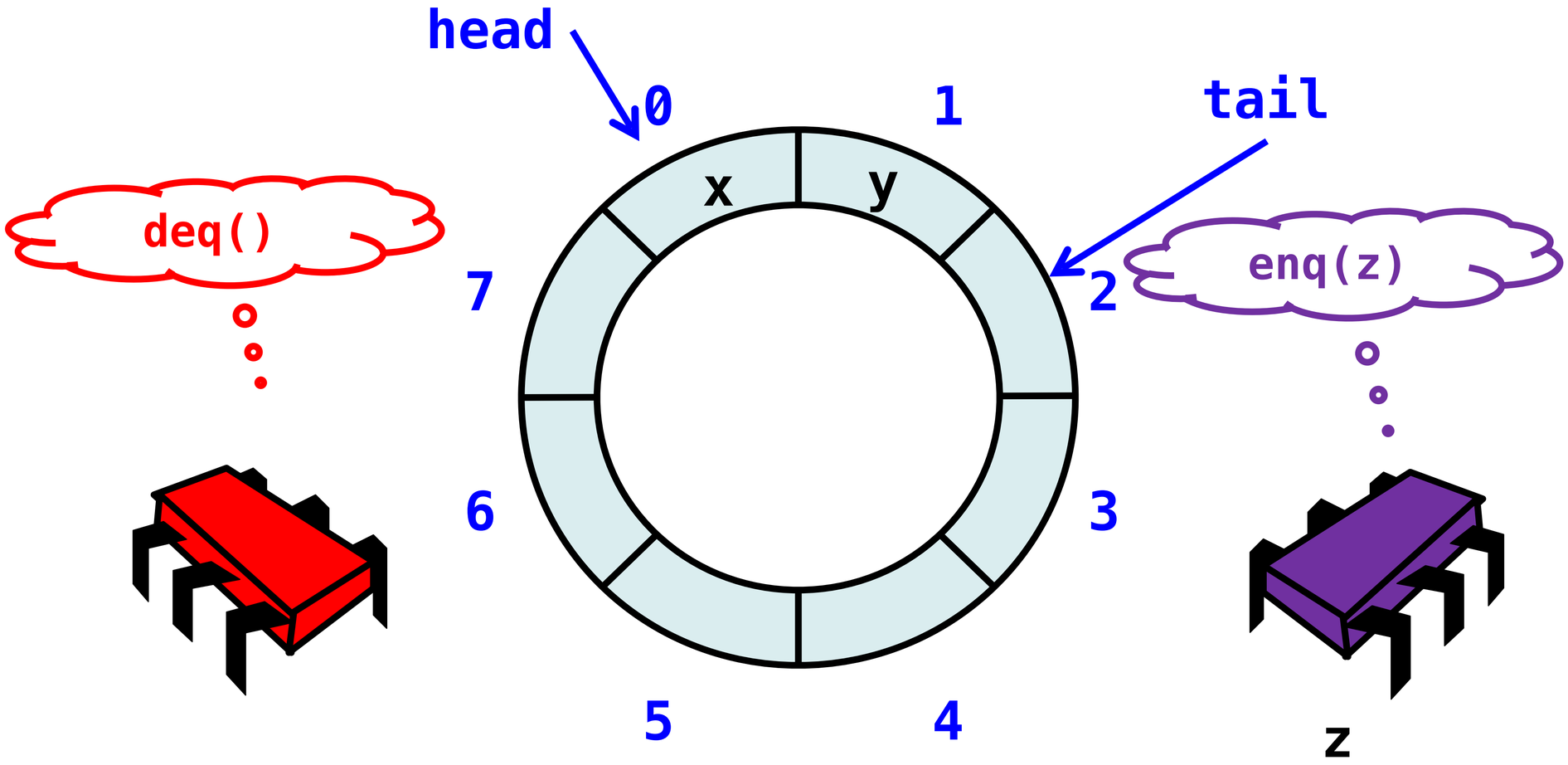

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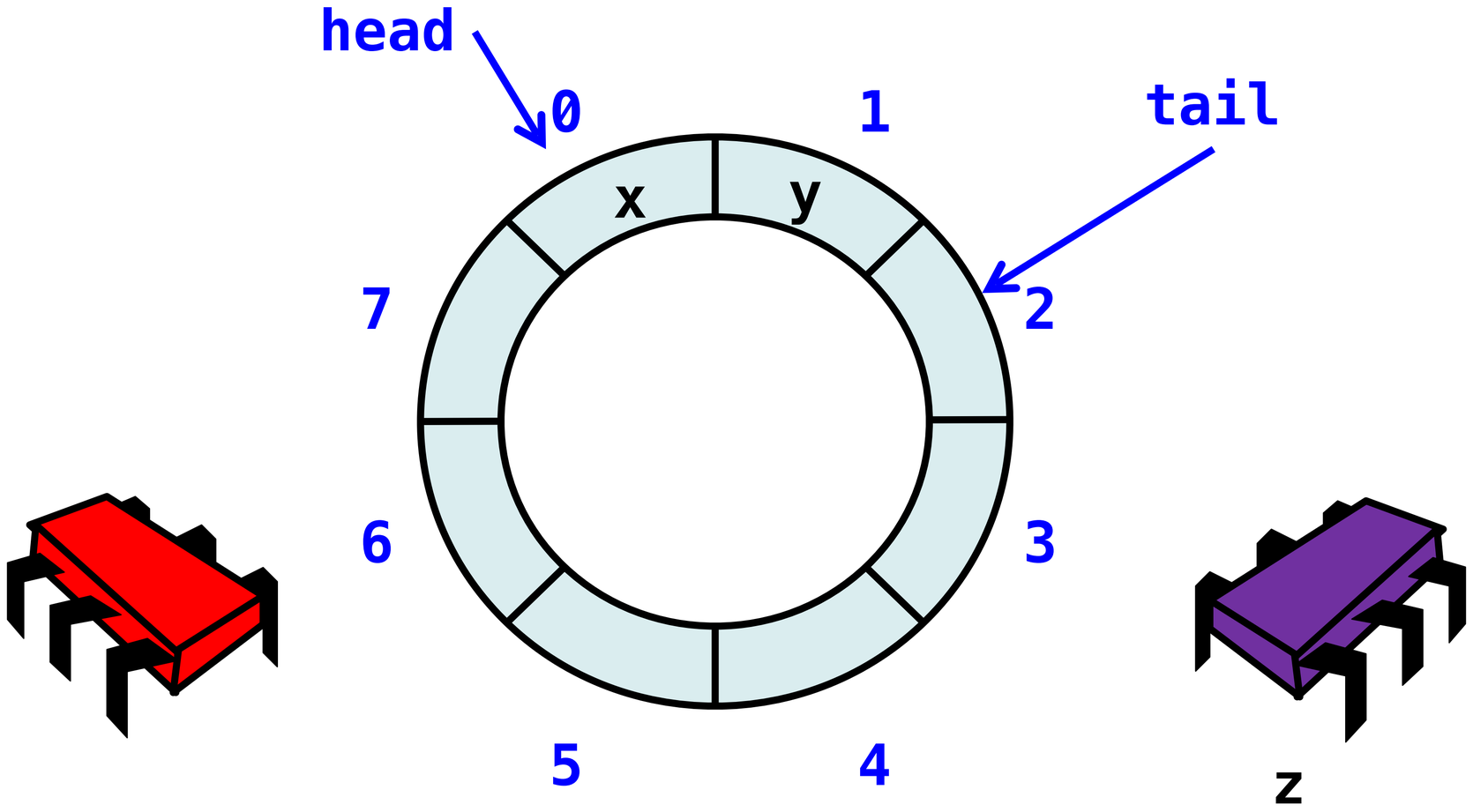

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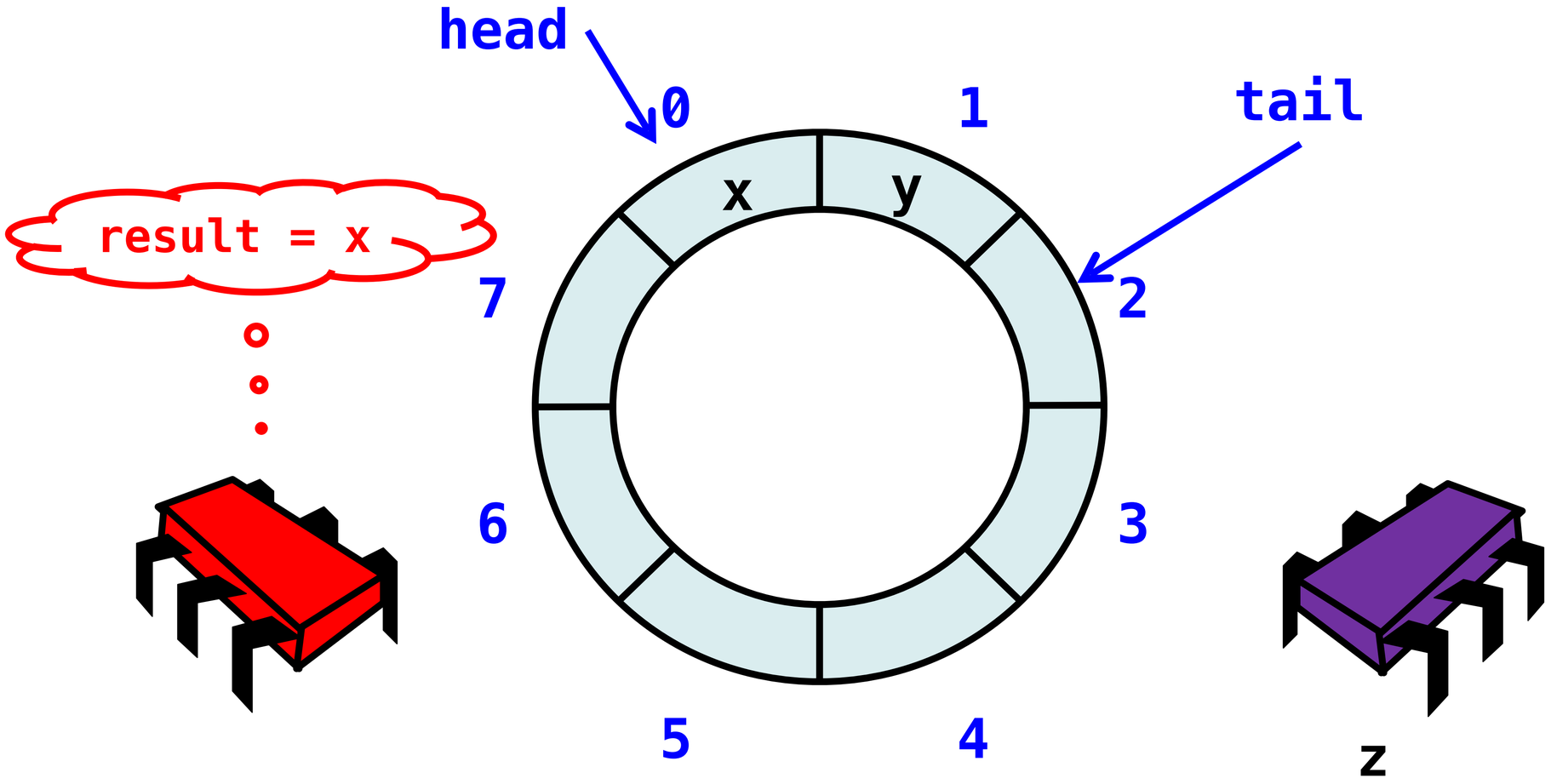

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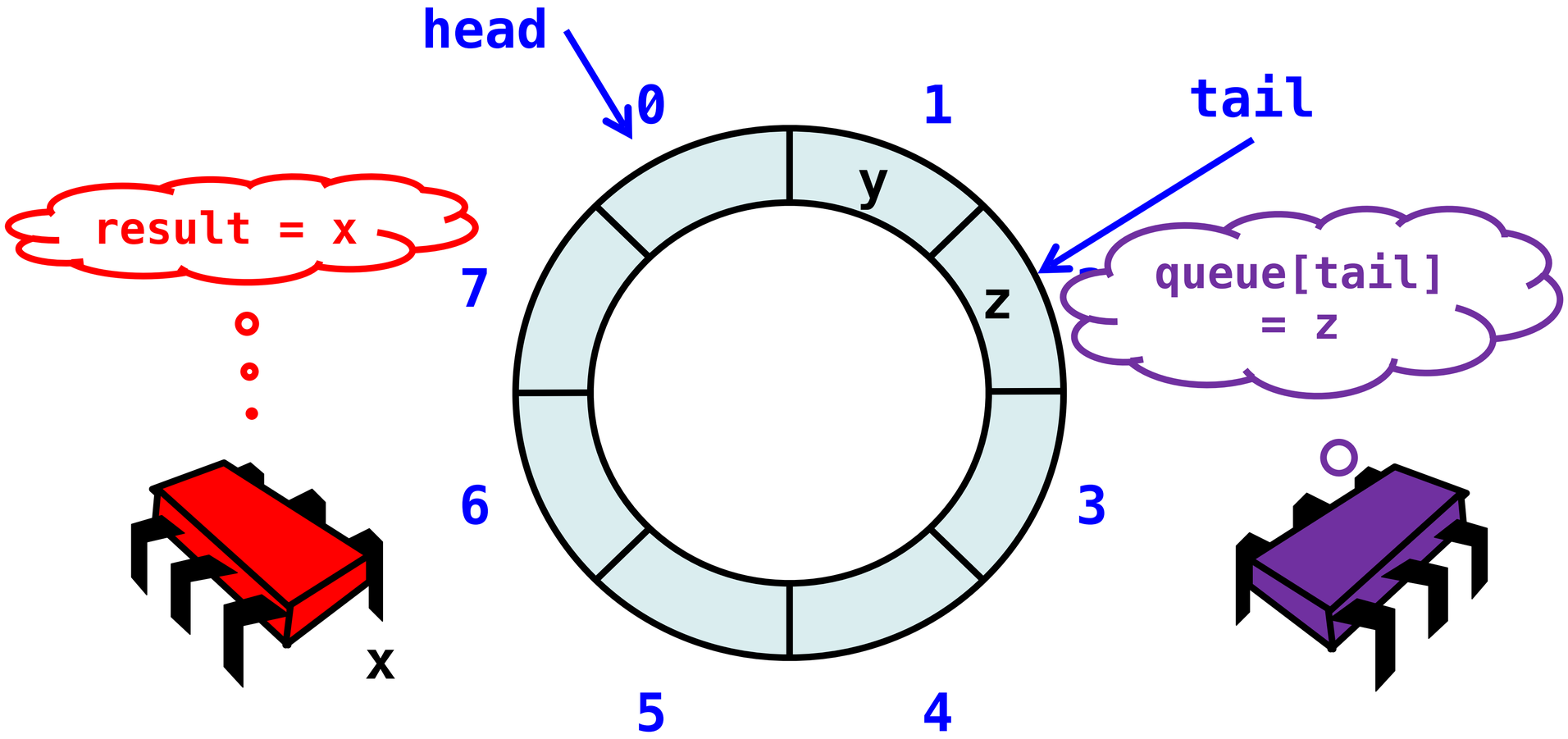

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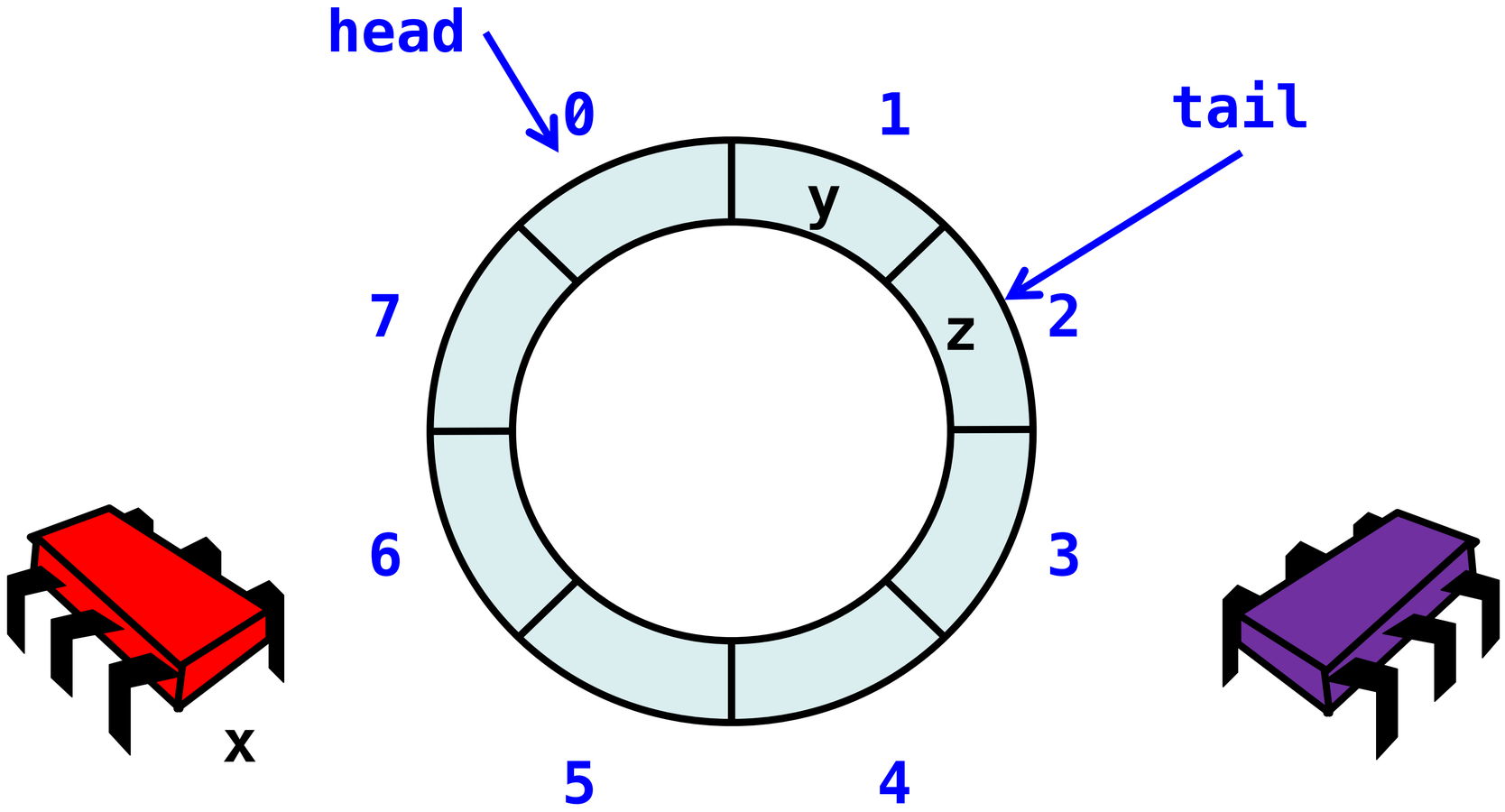

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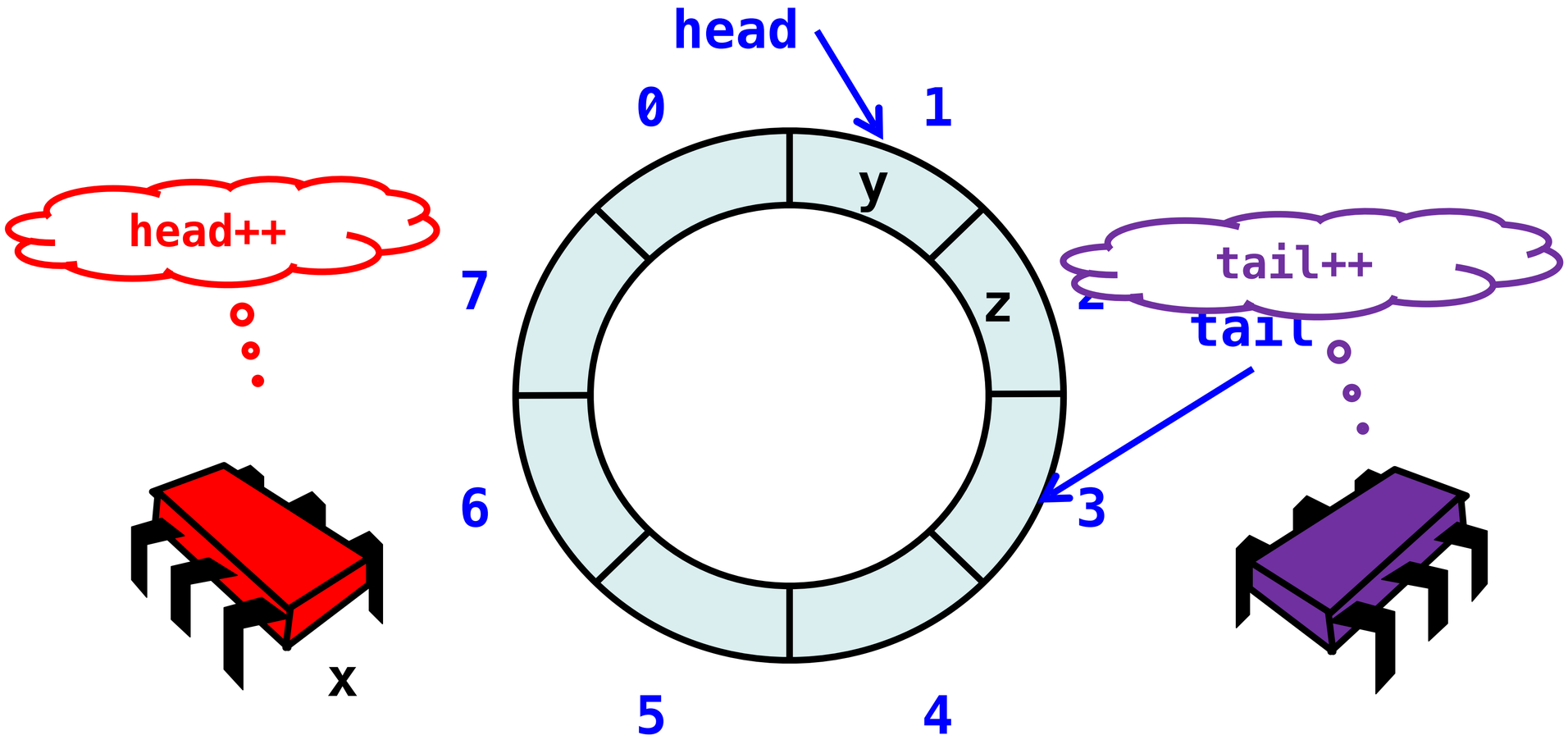

Wait-free 2-thread queue

No locks needed!

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(** Enqueue - should be called by only ONE thread *)
let enq q x =
  (* Check if queue is full *)
  if q.tail - q.head = q.capacity then
    raise Full;
  (* Write to the array *)
  q.items.(q.tail mod q.capacity) <- Some x;
  (* Advance tail *)
  q.tail <- q.tail + 1
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```
(** Dequeue - should be called by only ONE thread *)
let deq q =
  (* Check if queue is empty *)
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  (* Read from the array *)
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                            )
                            head + 1;
```

How do we define “correct” when modifications are not mutually exclusive?

Demo

Concurrency Specification

What *is* a concurrent queue?

- Need a way to ***specify*** a concurrent queue object
- Need a way to ***prove*** that an algorithm implements the object's specification
- Lets talk about object specifications …

Correctness and Progress

- In a concurrent setting, we need to specify both the ***safety*** and the ***liveness*** properties of an object
- Need a way to define
  - when an implementation is ***correct***
  - the conditions under which it guarantees ***progress***

Correctness and Progress

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  - when an implementation is ***correct***
  - the conditions under which it guarantees ***progress***

Lets begin with correctness

Sequential Objects

- Each object has a ***state***
  - Usually given by a set of ***fields***
  - Queue example: sequence of items
- Each object has a set of ***methods***
  - Only way to manipulate state
  - Queue example: **`enq`** and **`deq`** methods

Sequential Specifications

- If (***precondition***)
  - the object is in such-and-such a state
  - before you call the method,

Sequential Specifications

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  - before you call the method,
- Then (***postcondition***)
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Sequential Specifications

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  - the object is in such-and-such a state
  - before you call the method,
- Then (***postcondition***)
  - the method will return a particular value
  - or throw a particular exception.
- and (***postcondition, cont***)
  - the object will be in some other state
  - when the method returns,

Pre and Post Conditions for Dequeue

- Precondition:
  - Queue is ***non-empty***
- Postcondition:
  - Returns first item in queue
- Postcondition:
  - Removes first item in queue

Pre and Post Conditions for Dequeue

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- Precondition:
  - Queue is ***empty***
- Postcondition:
  - Raises `Empty` exception
- Postcondition:
  - Queue state is unchanged

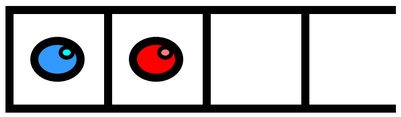
Why Sequential Specifications Totally Rock

- **Interactions among *methods* captured by side-effects on object state**
  - State meaningful between method calls
- ***Documentation* size is linear in the number of methods**
  - Each method described in isolation
- **Can add *new methods***
  - Without changing descriptions of old methods

What about concurrent Specifications?

- Methods?
- Documentation?
- Adding new methods?

Methods take time



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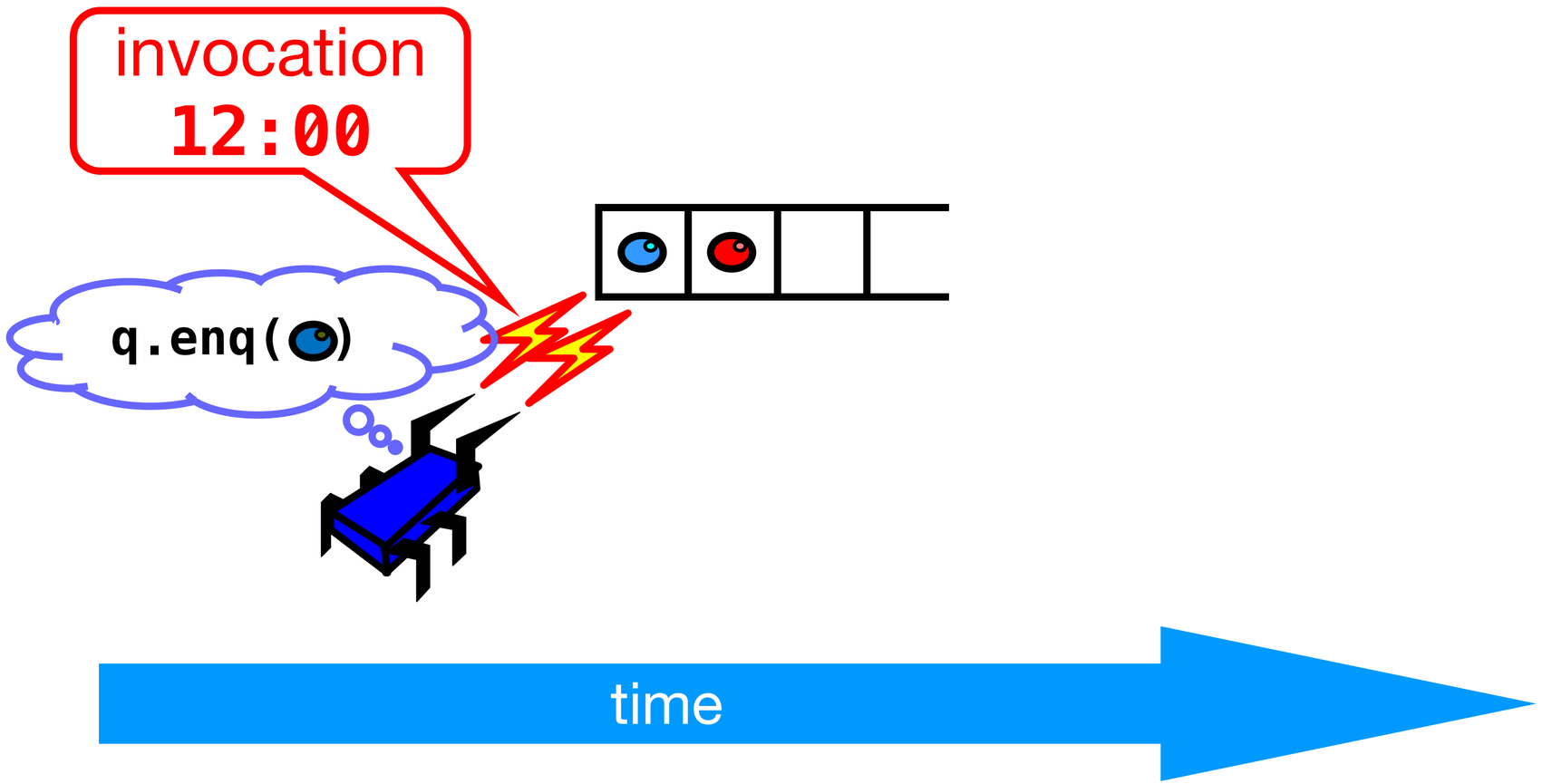

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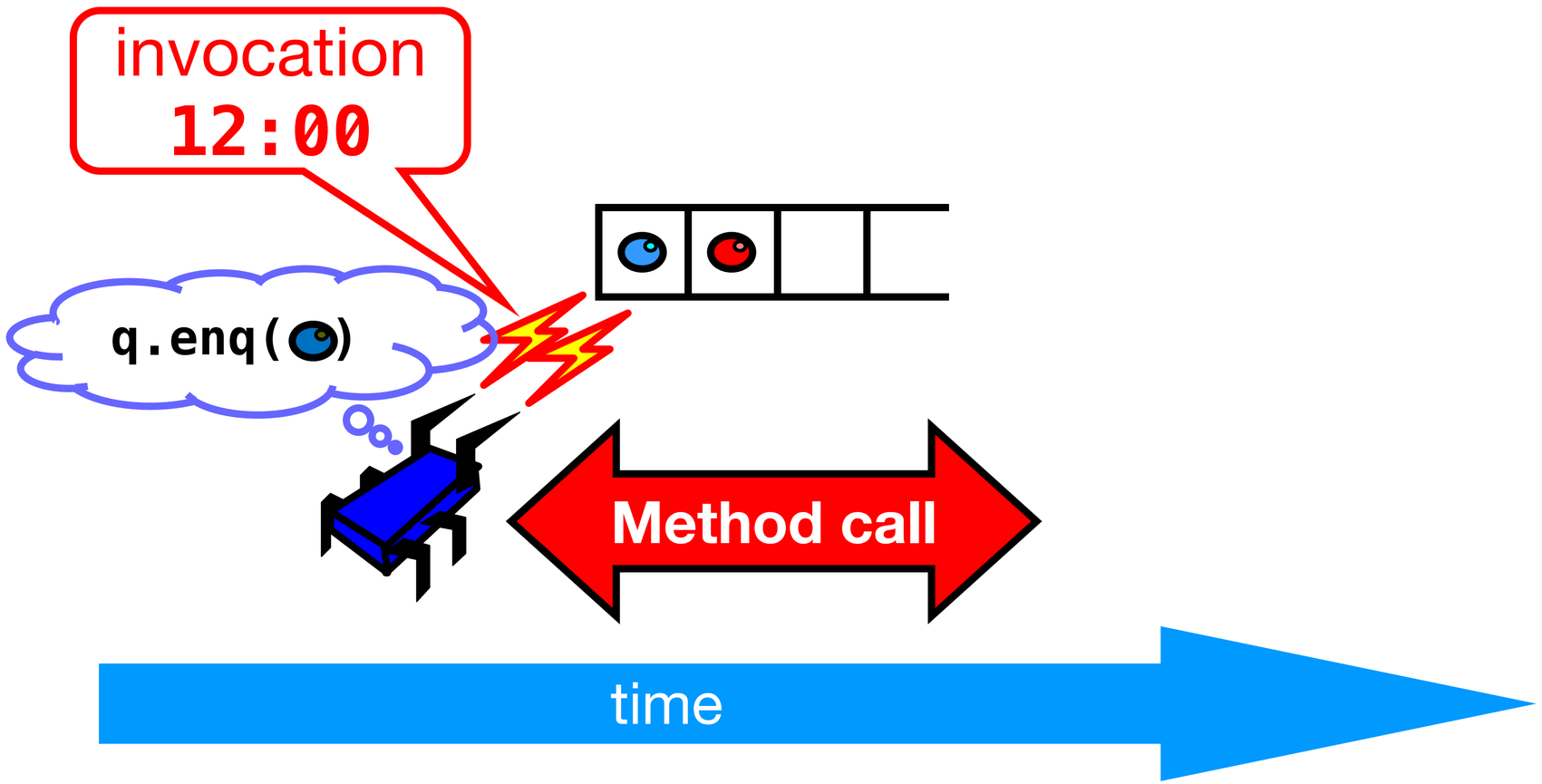

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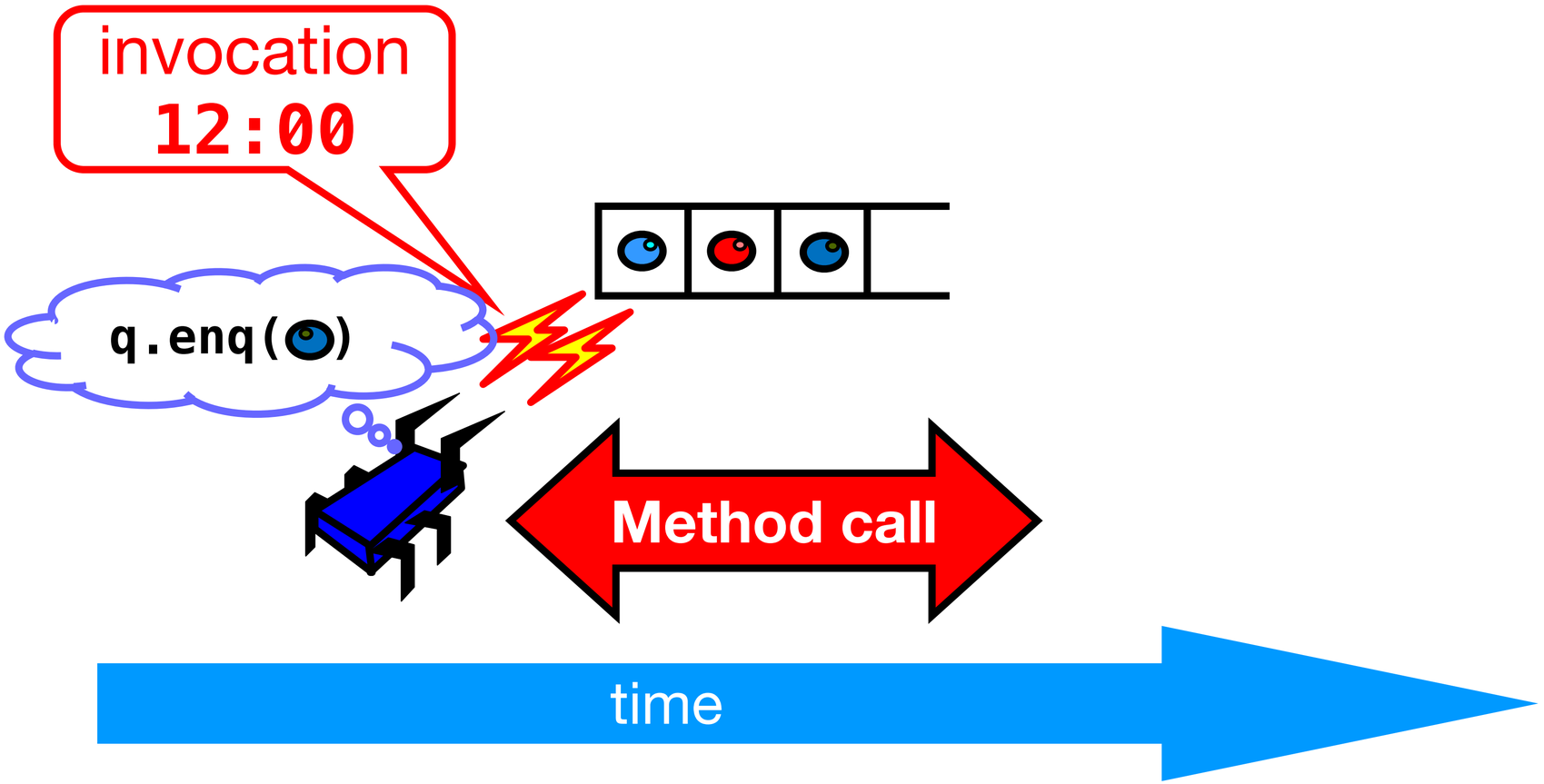

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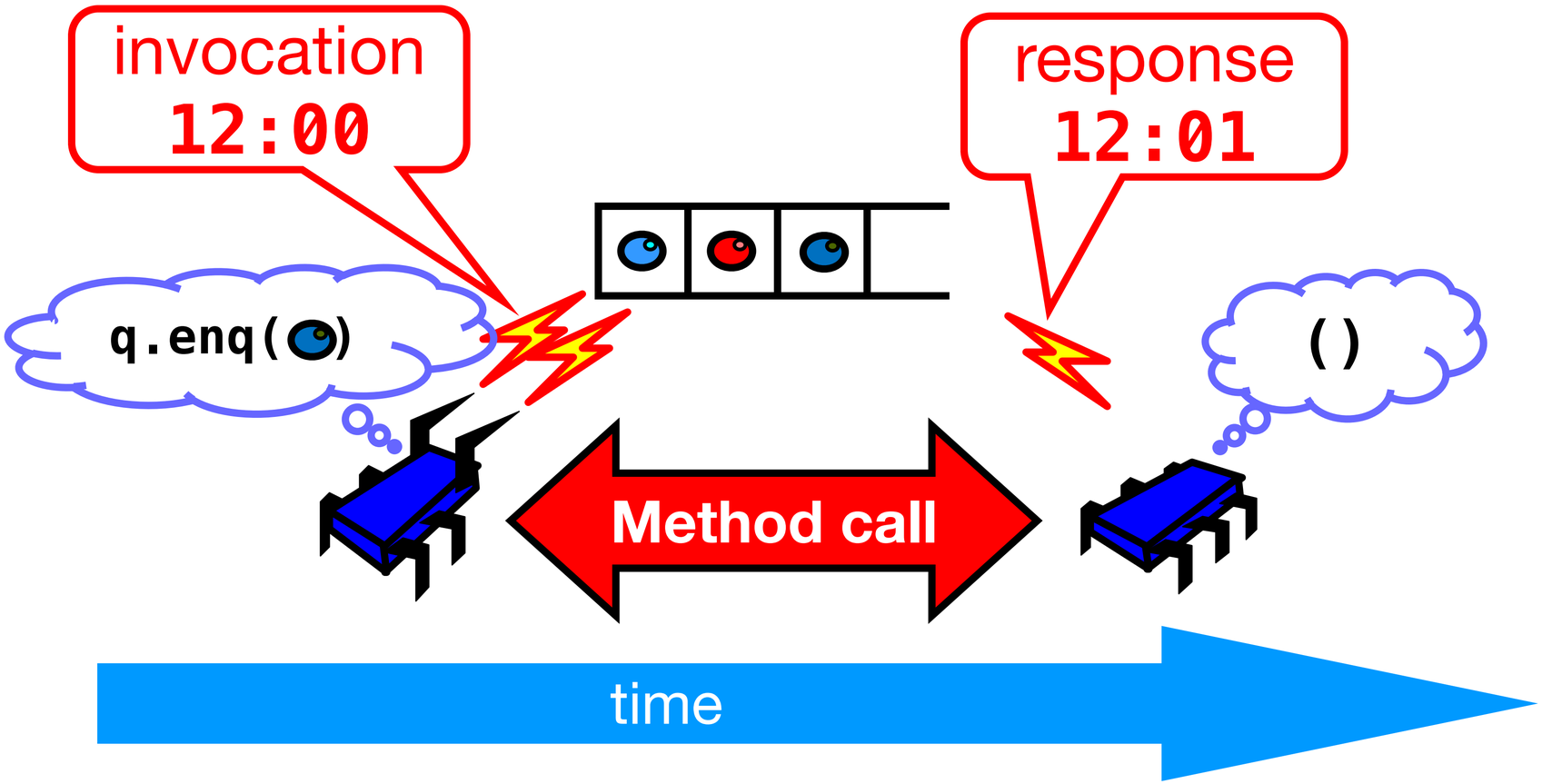
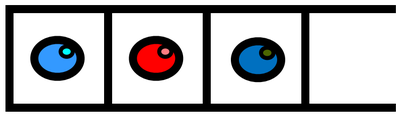


Sequential vs Concurrent

- Sequential
  - Methods take time? Who knew?
- Concurrent
  - Method call is not an ***event***
  - Method call is an ***interval***

Concurrent Methods Take Overlapping Time



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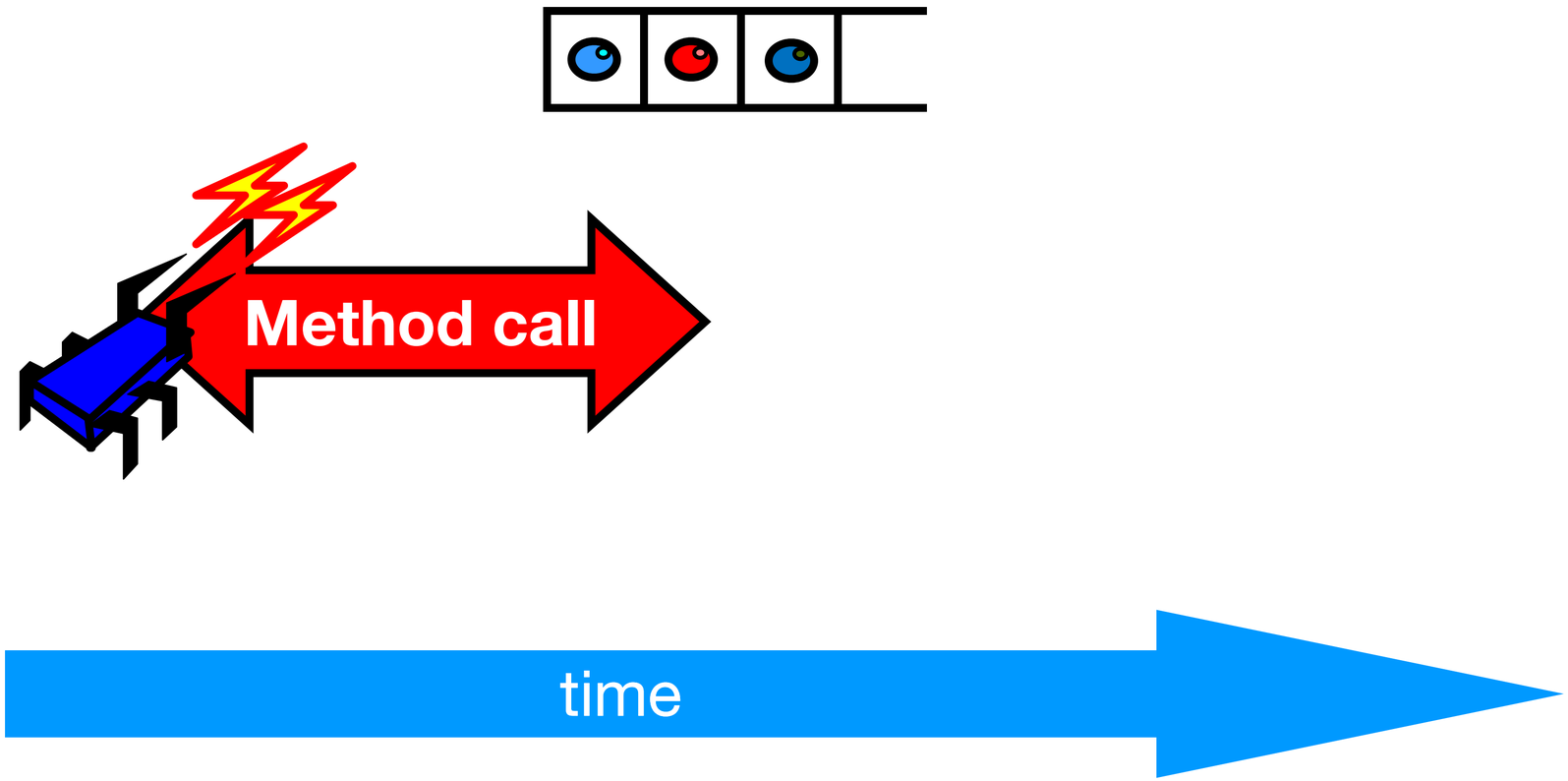

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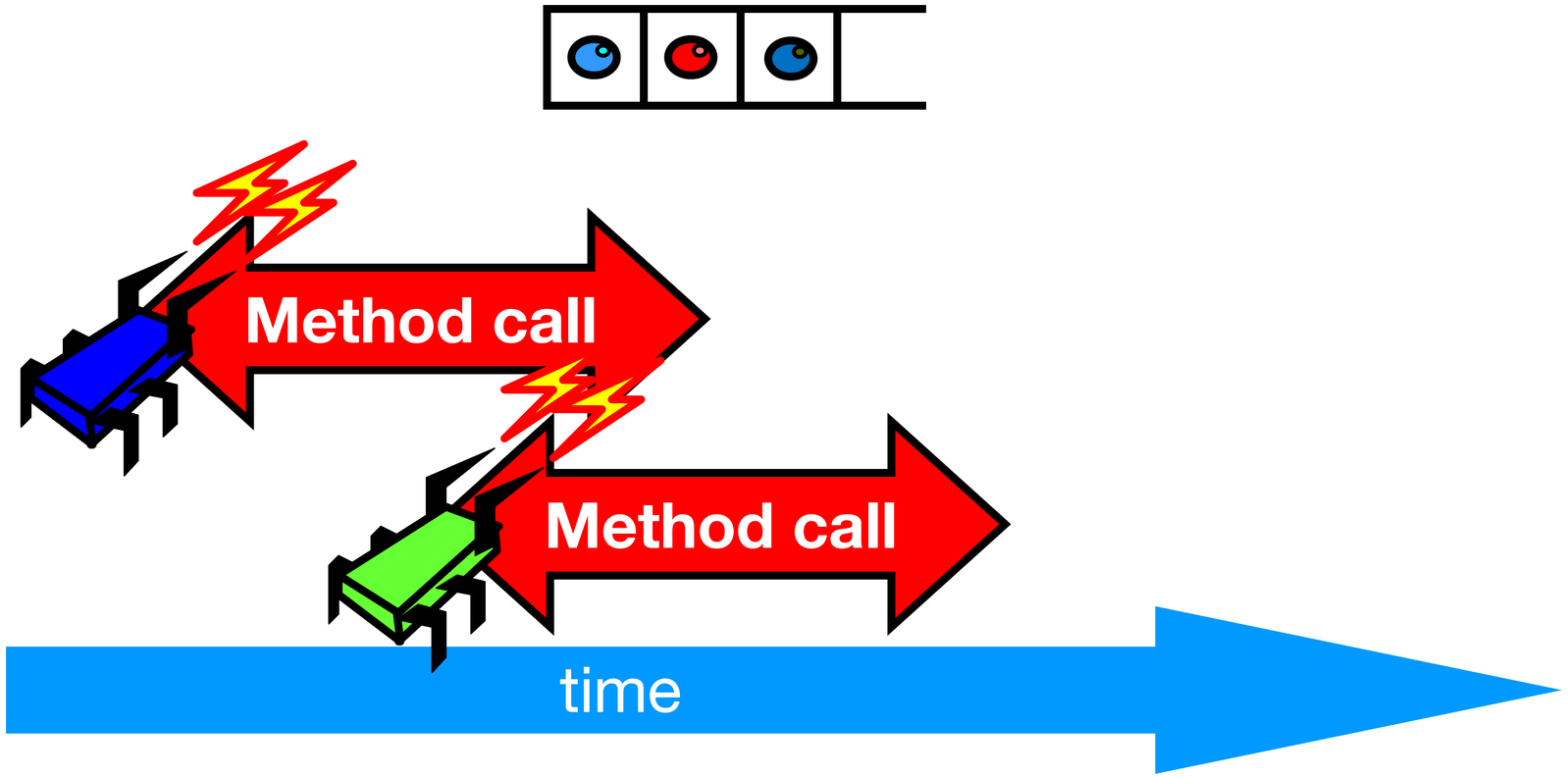

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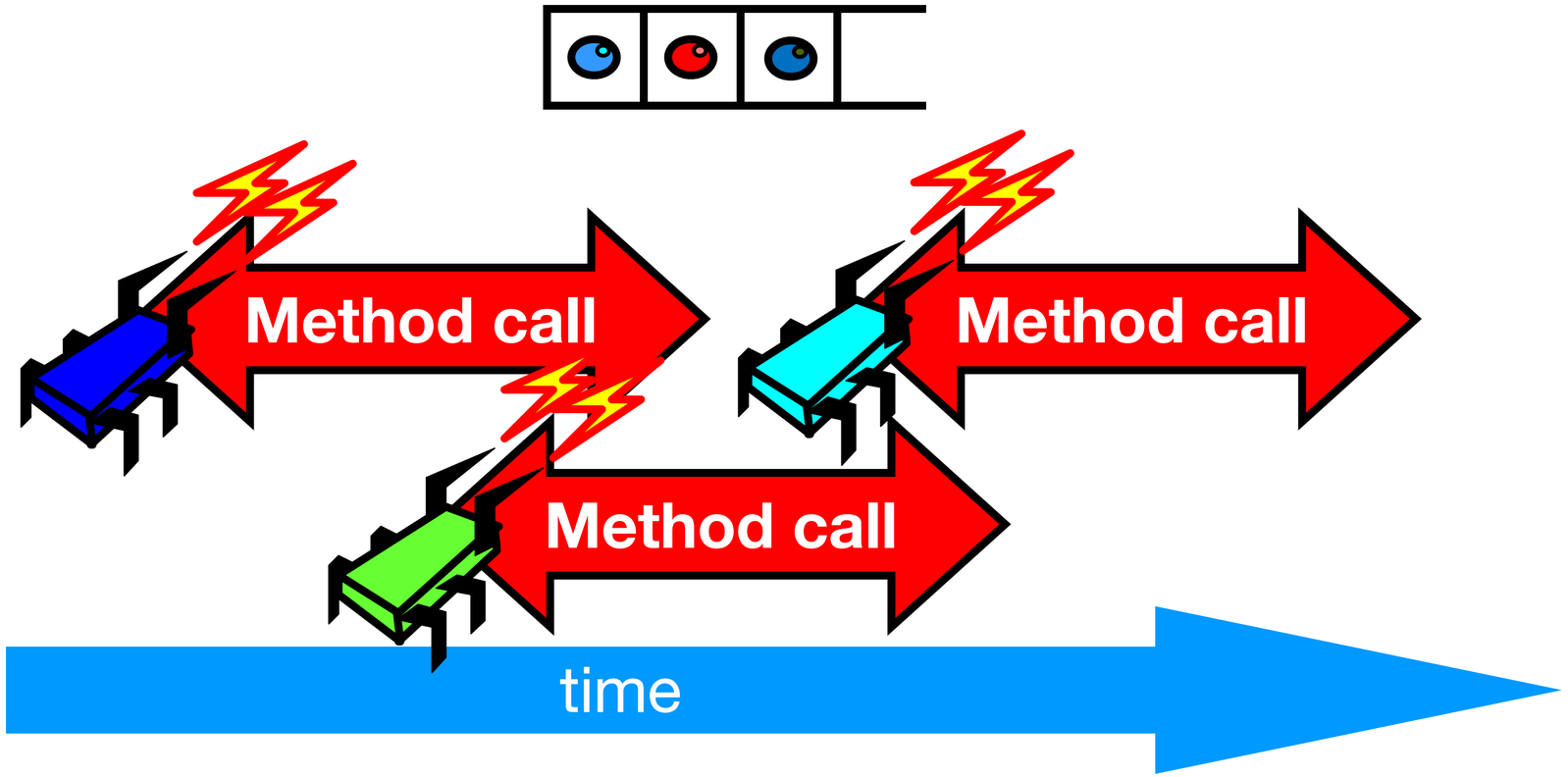

Sequential vs Concurrent

- Sequential
  - Object needs a meaningful state only ***between*** method calls
- Concurrent
  - Because method calls overlap, the object might ***never*** be between method calls

Sequential vs Concurrent

- Sequential:
  - Each method described in isolation
- Concurrent
  - Must characterize ***all*** possible interactions with concurrent calls
    - What if two **`enq()`** calls overlap?
    - Two **`deq()`** calls? **`enq()`** and **`deq()`**? …

Sequential vs Concurrent

- Sequential:
  - Can add new methods without affecting older methods
- Concurrent:
  - Everything can potentially interact with everything else

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The Big Question

- What does it *mean* for a ***concurrent*** object to be correct?
  - What ***is*** a concurrent FIFO queue?
  - FIFO means ***strict temporal order***
  - Concurrent means ***ambiguous temporal order***

Intuitively

```
let deq q =
  Mutex.lock q.lock;
  try
    if q.tail = q.head then
      raise Empty;
    match q.items.(q.head mod q.capacity) with
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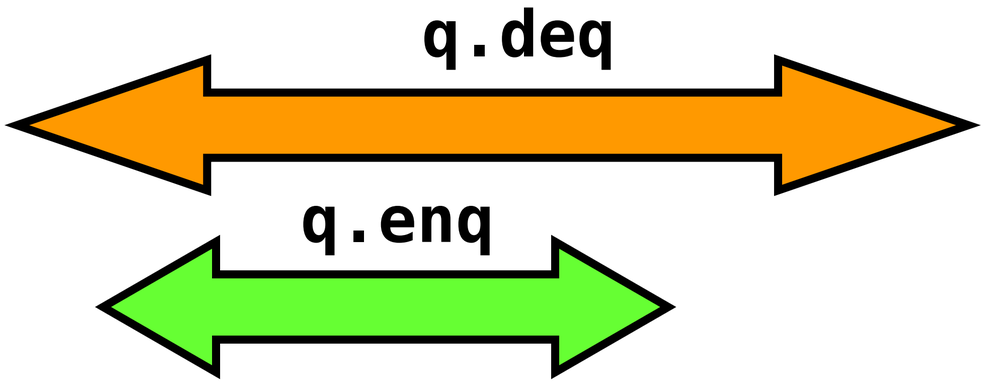
All queue modifications are mutually exclusive

Intuitively



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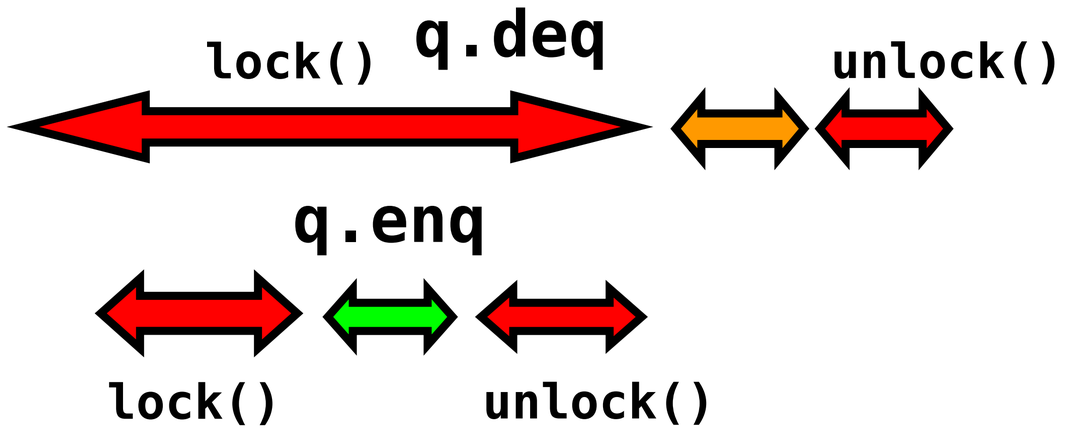

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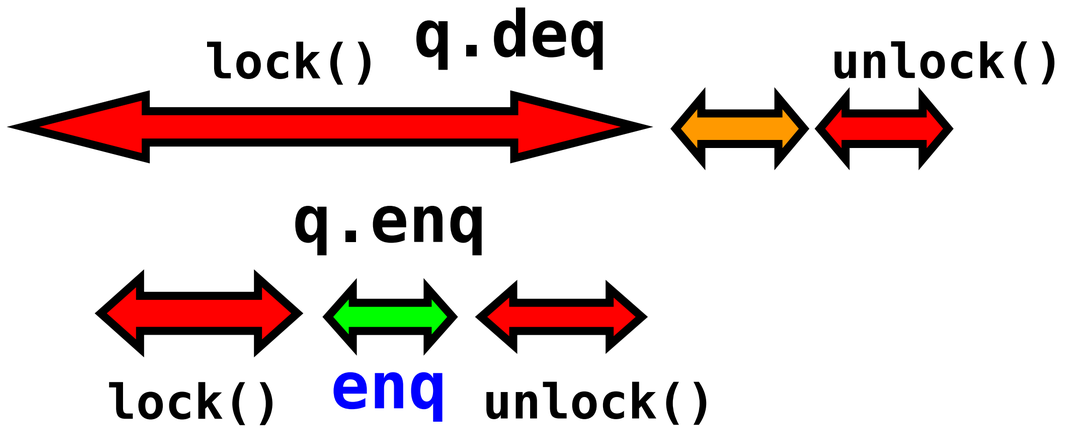

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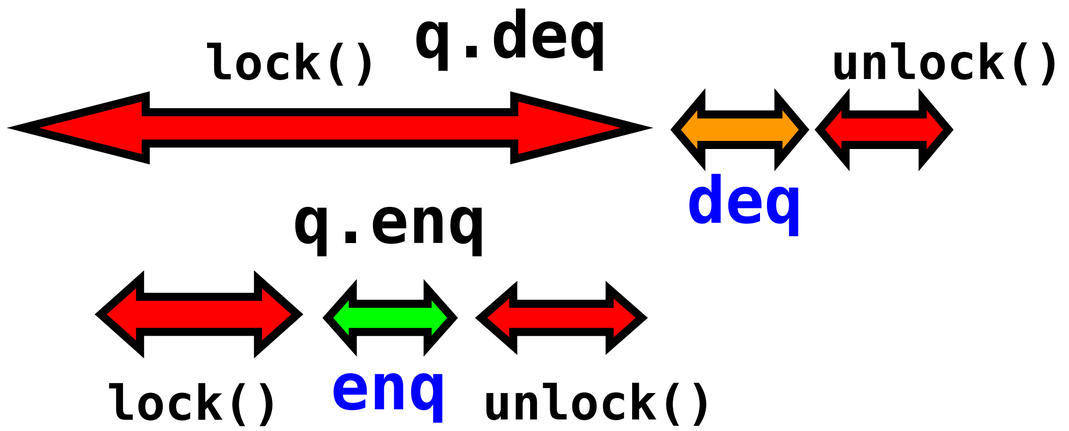
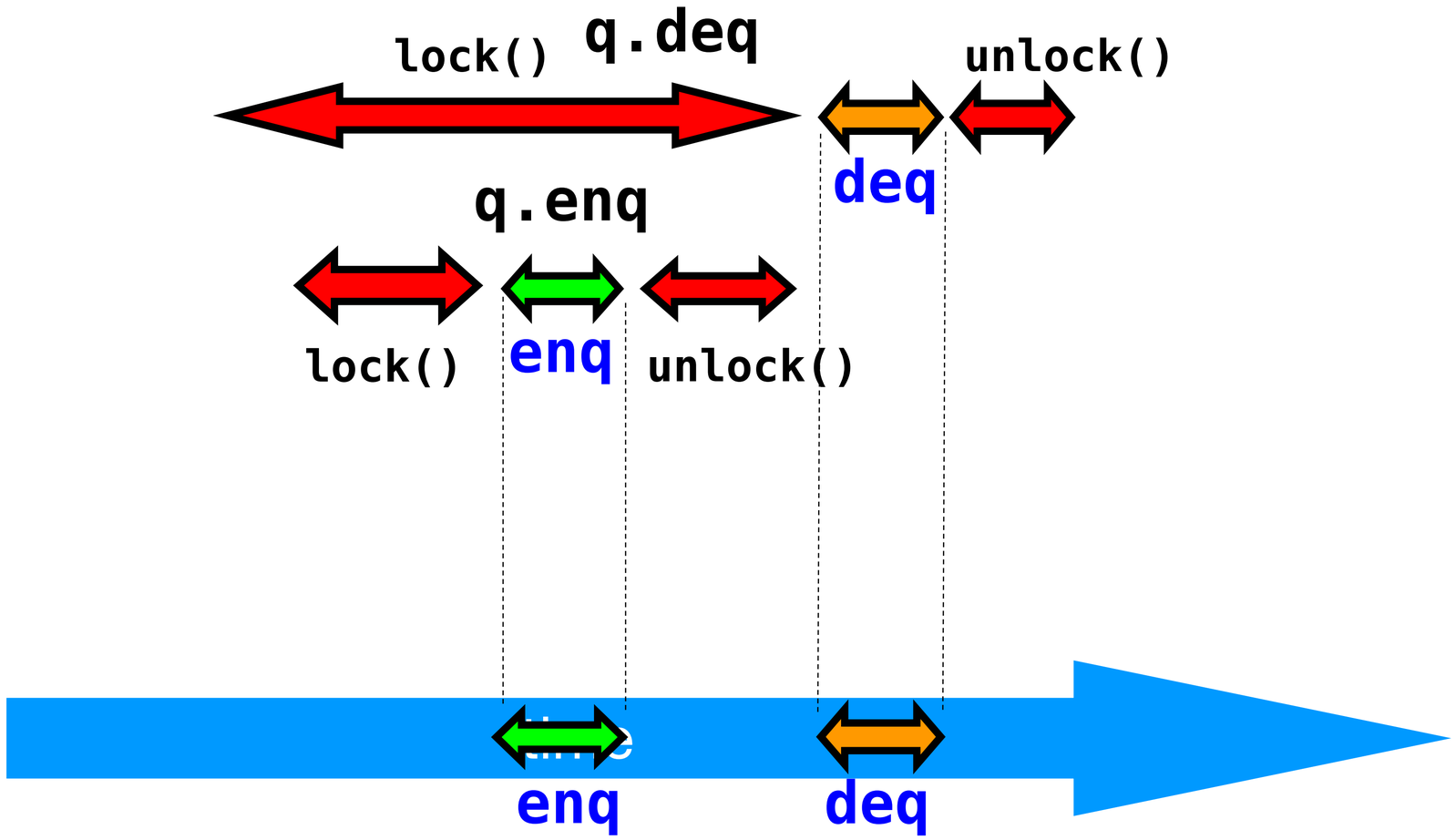
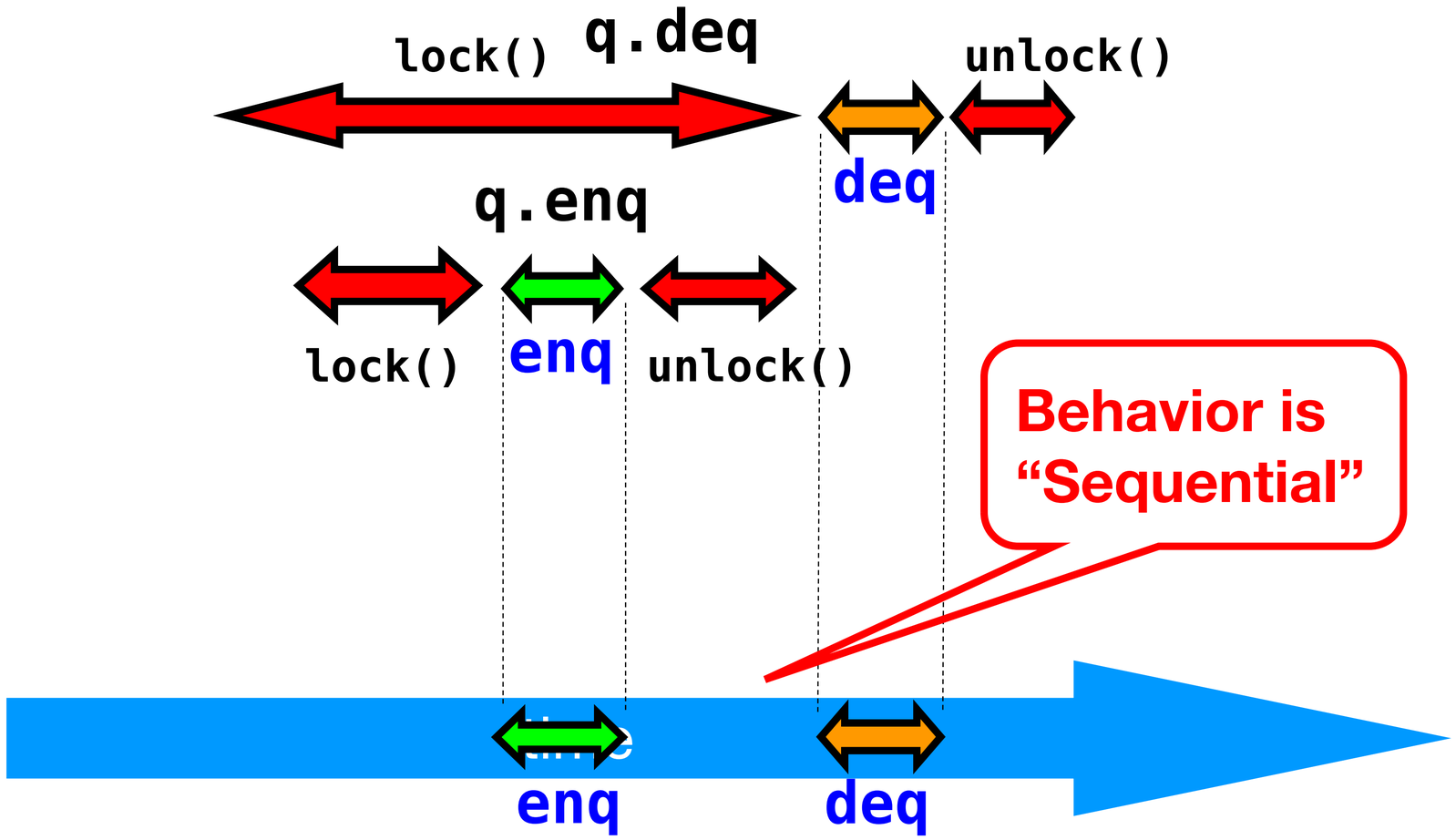


Intuitively

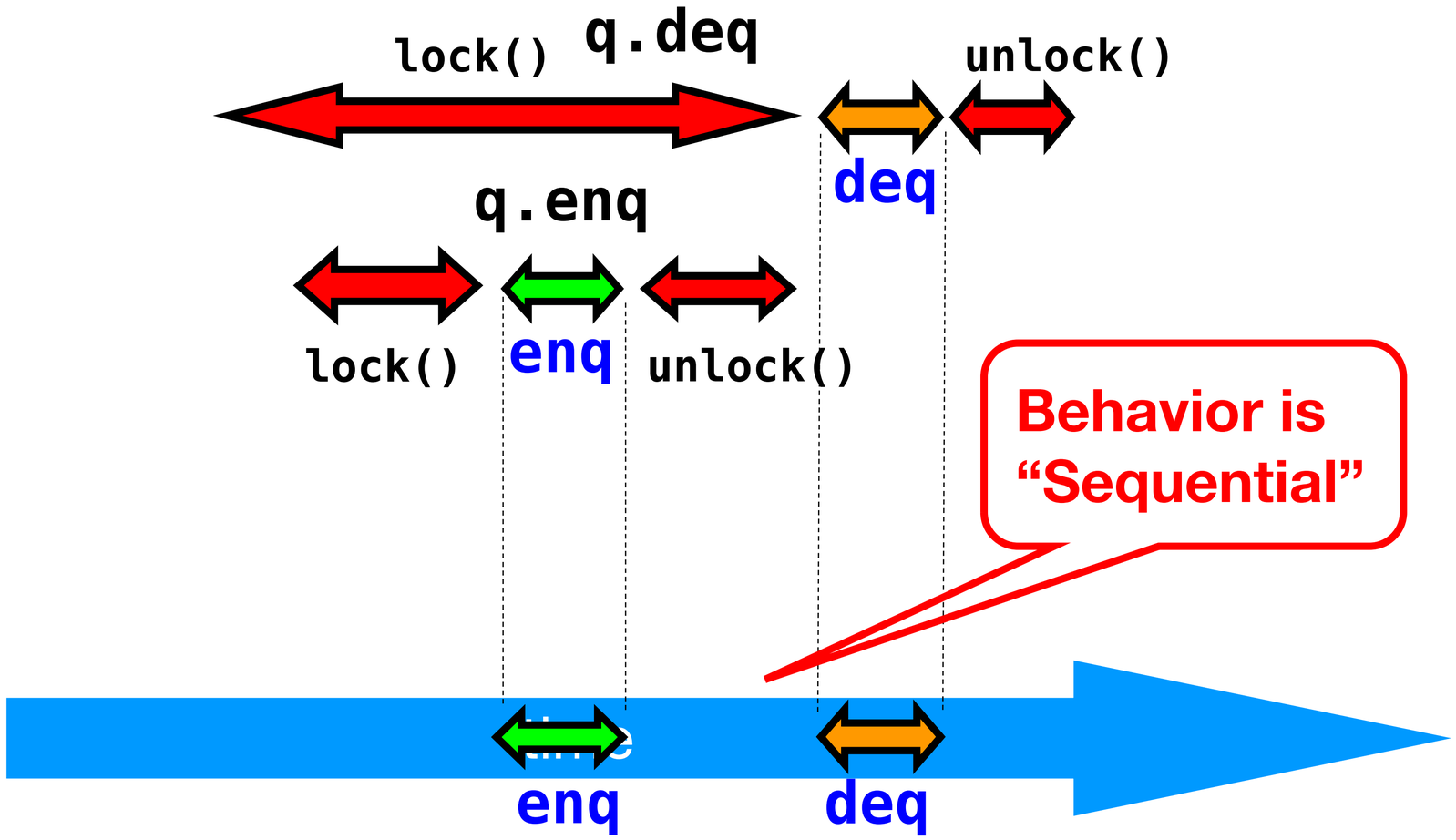


Intuitively



Intuitively

Lets capture the idea of describing the concurrent via the sequential



Linearizability

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  - Instantaneously
  - Between ***invocation*** and ***response*** events

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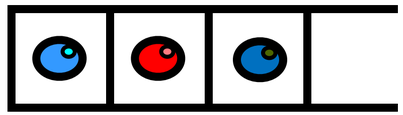
- Each method should
  - “take effect”
  - Instantaneously
  - Between ***invocation*** and ***response*** events
- Object is correct if this “sequentialised” behaviour is correct
- Any such concurrent object is
  - **Linearizable™**

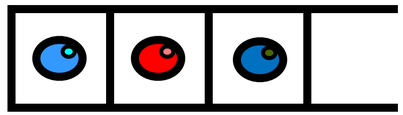
Is it really about the object?

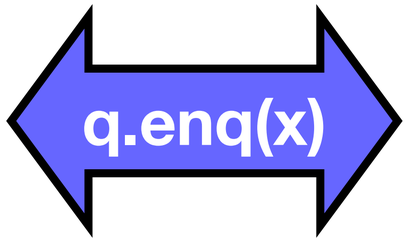
- Each method should
  - “take effect”
  - Instantaneously
  - Between ***invocation*** and ***response*** events
- Sounds like a property of *an execution…*
- A linearizable *object*
  - One of whose all possible executions are linearizable

Example



Example





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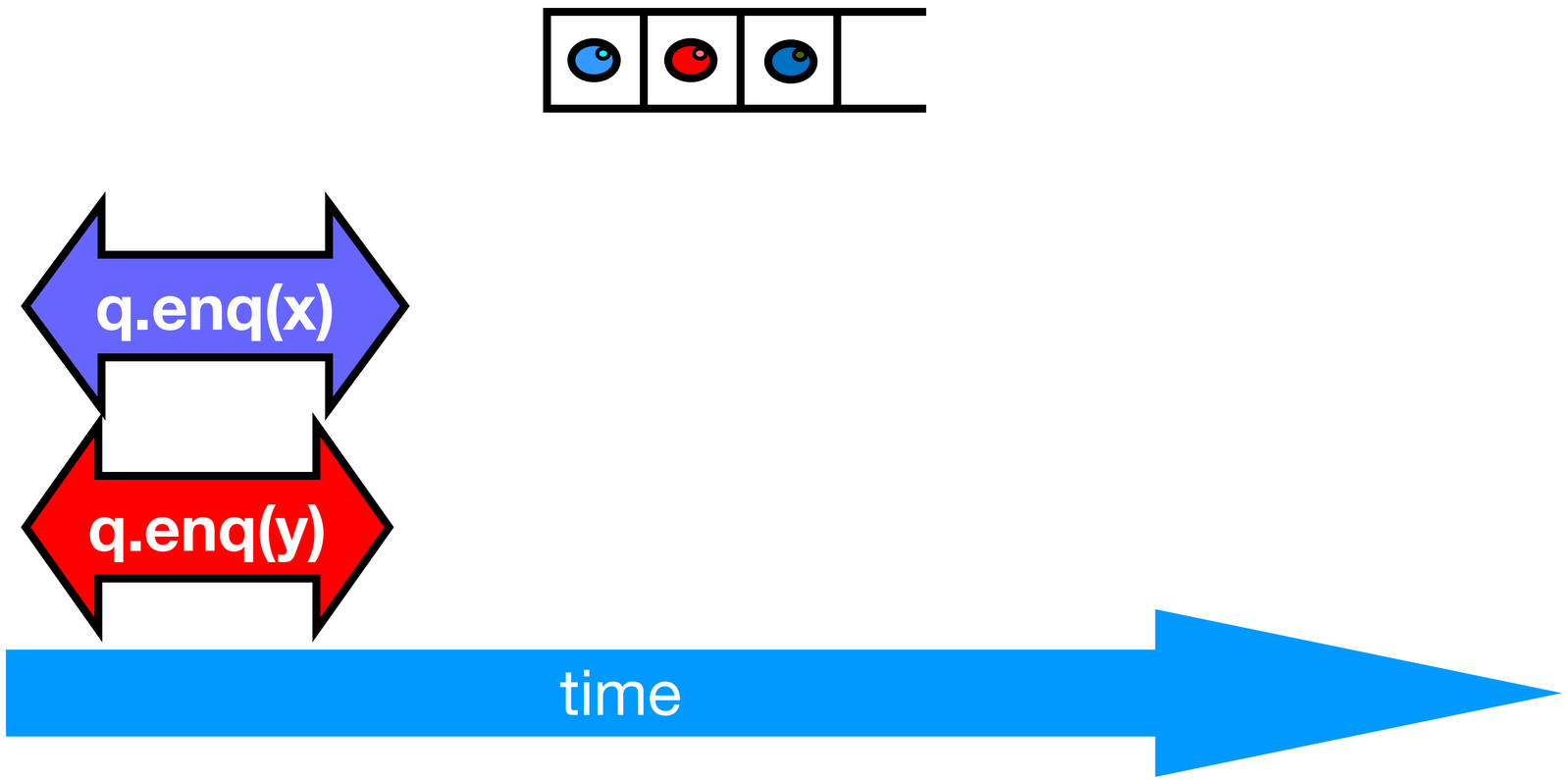

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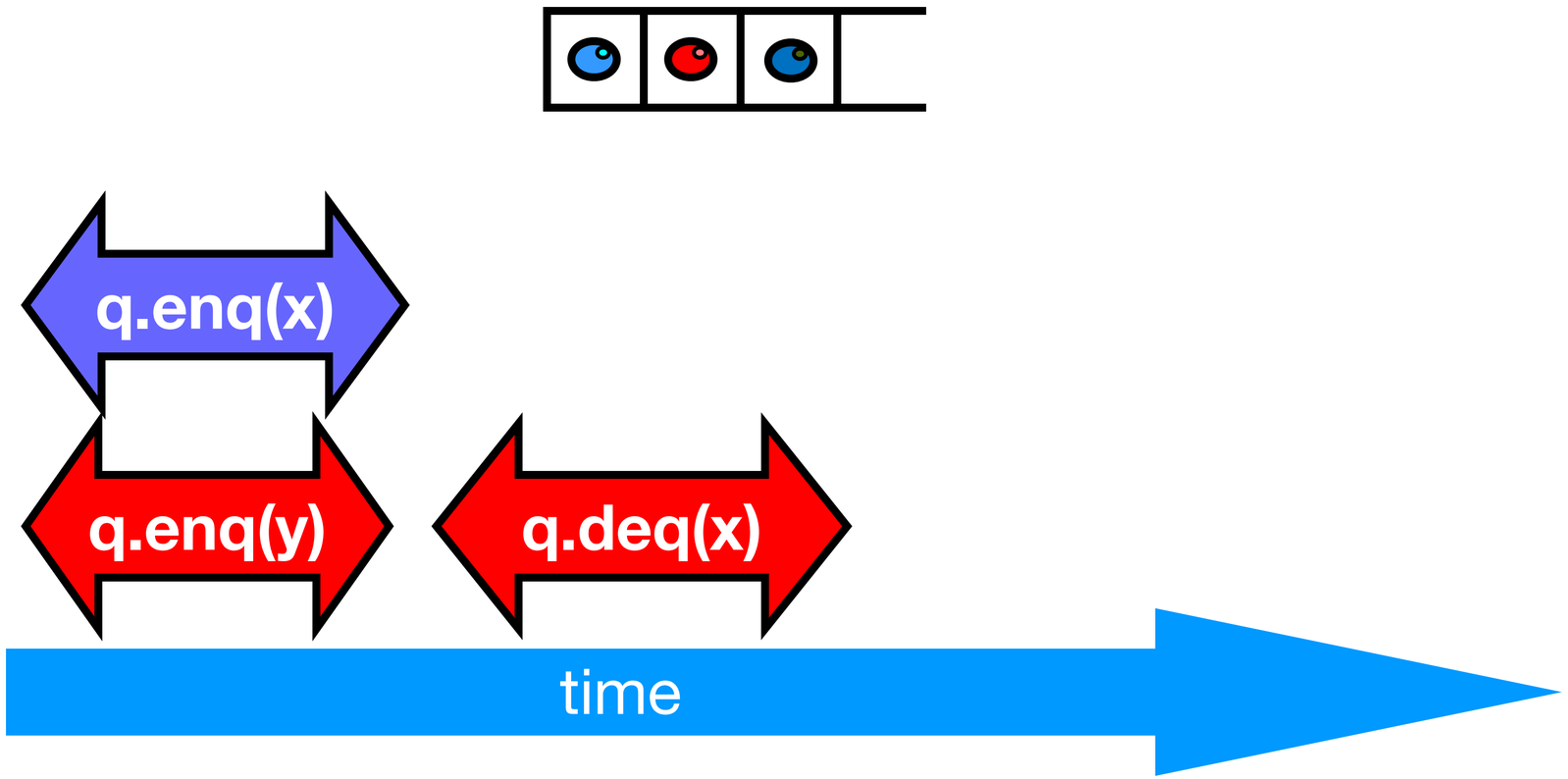

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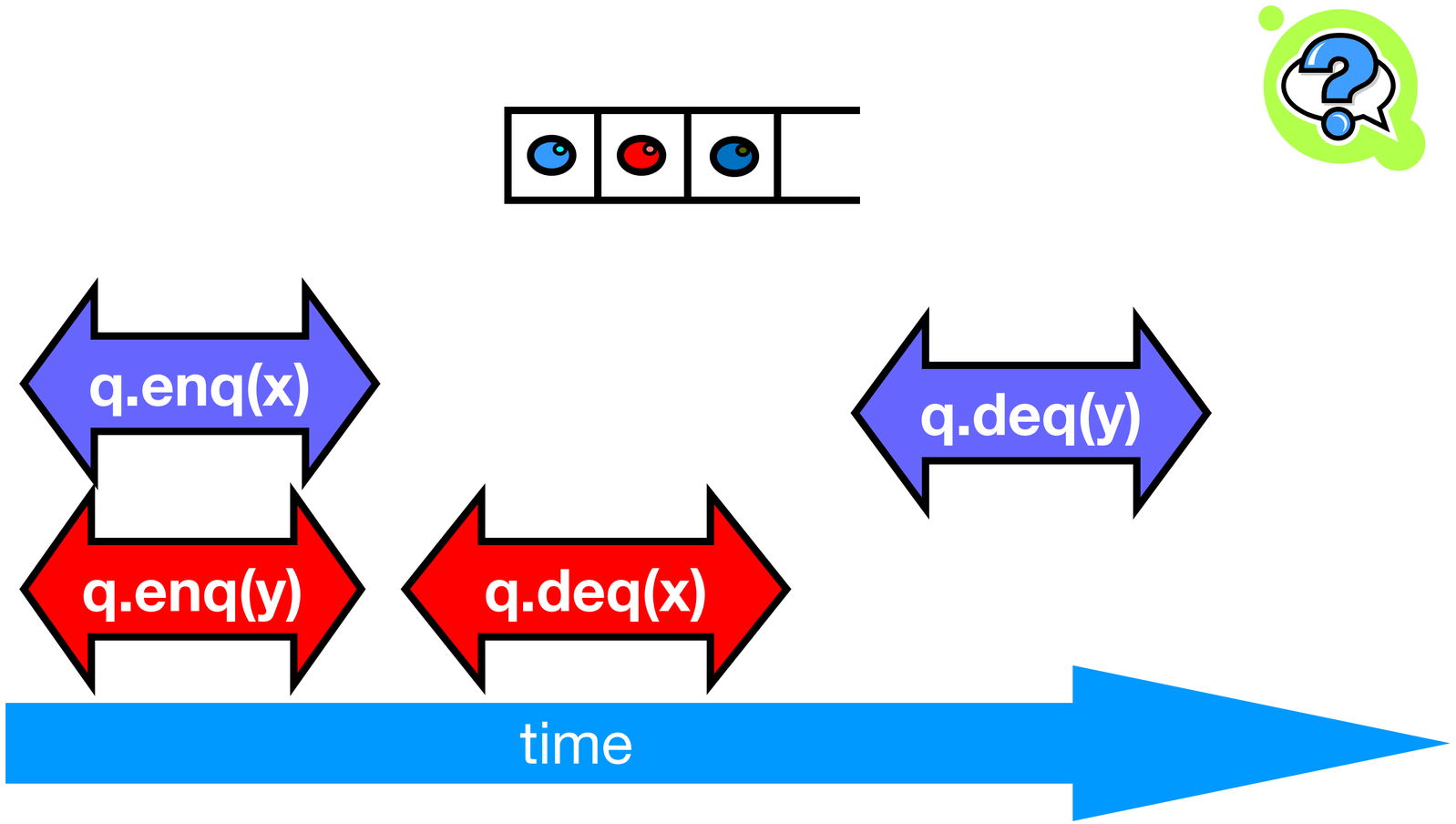

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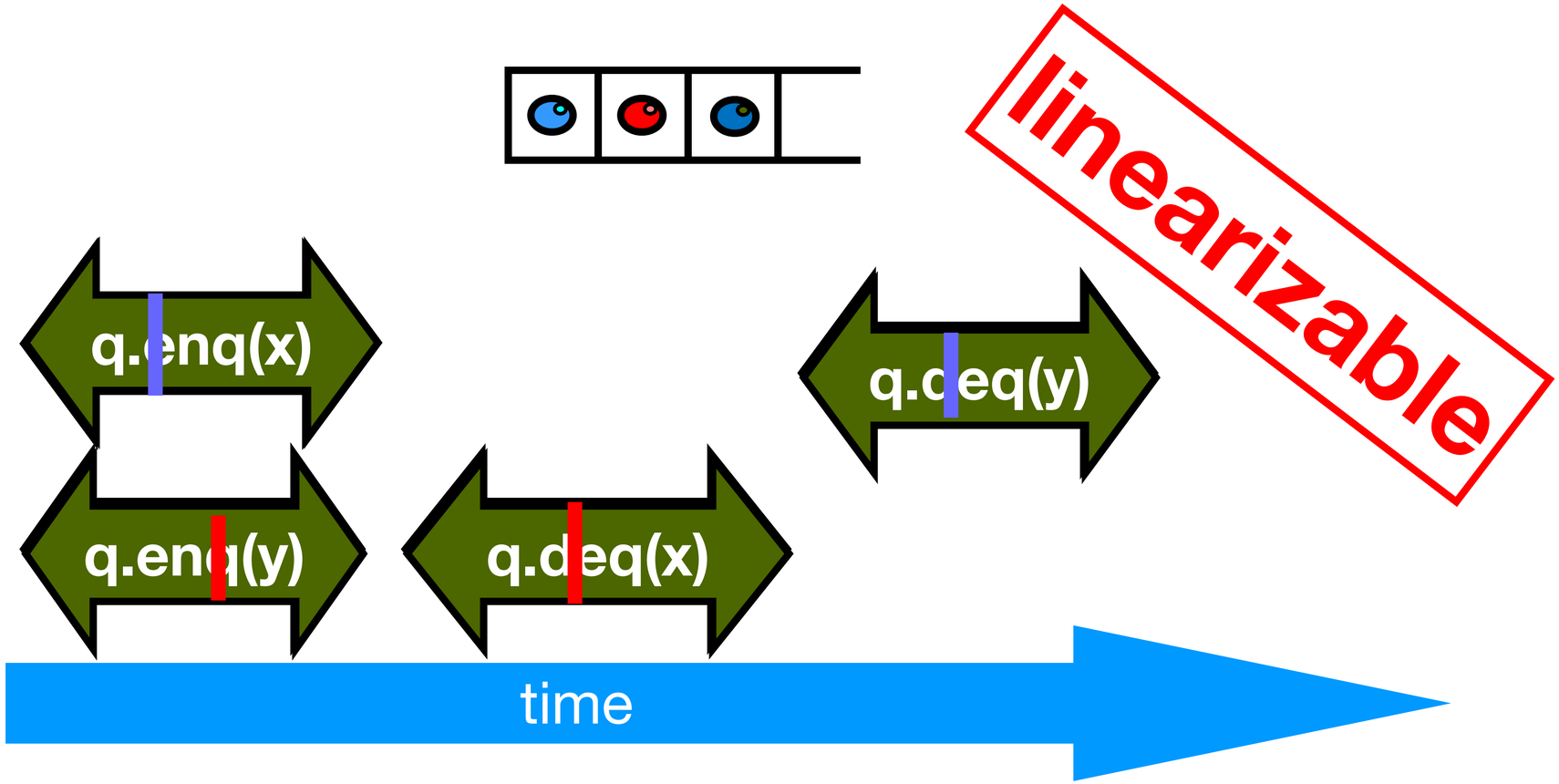

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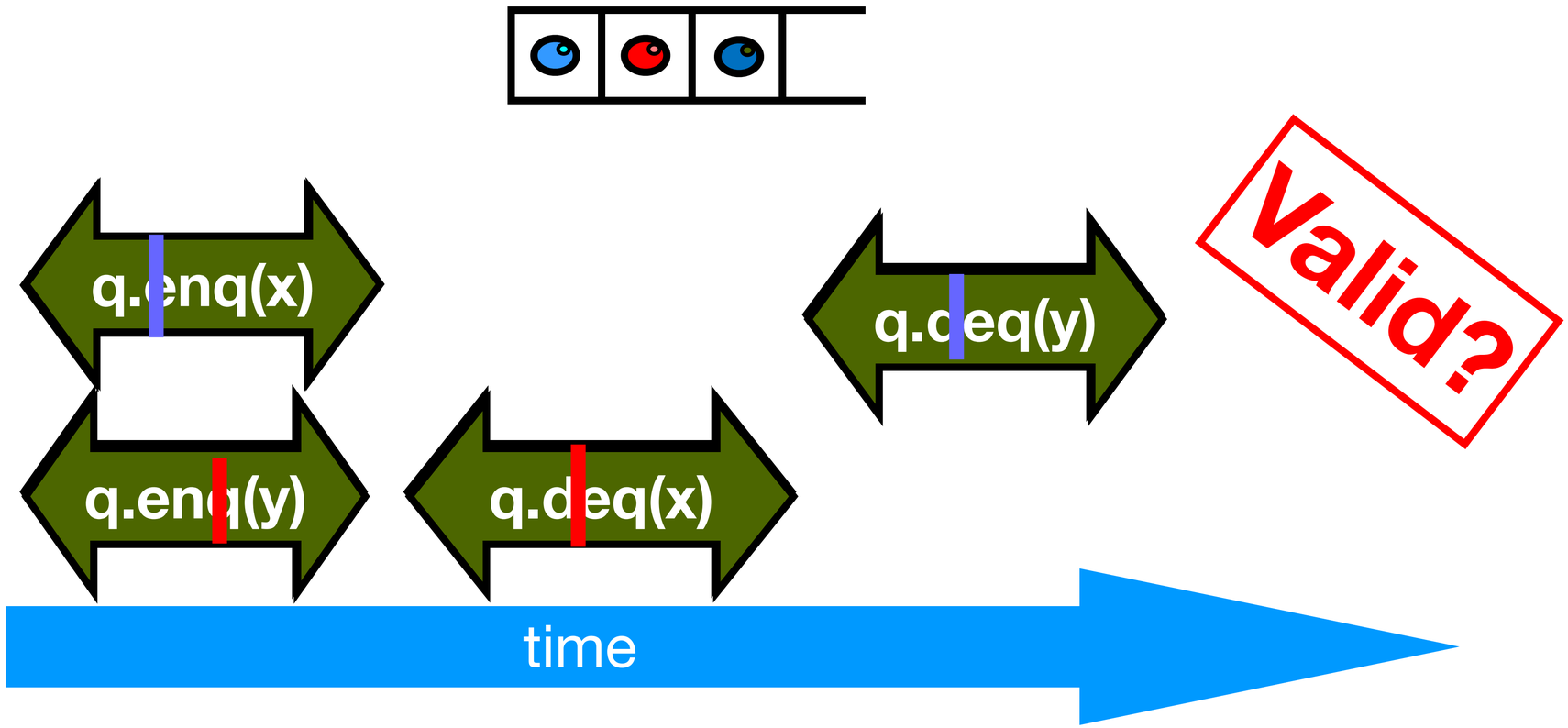
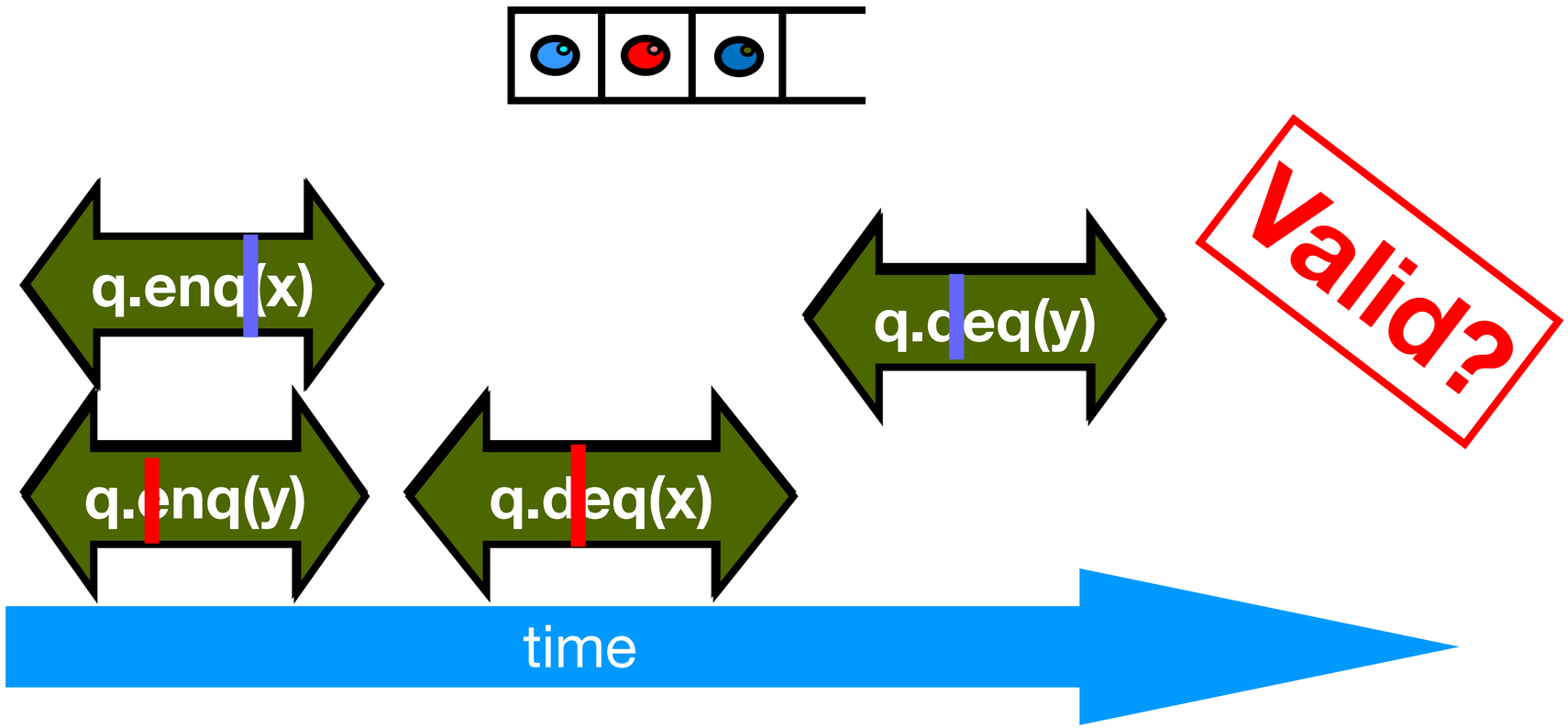


Example



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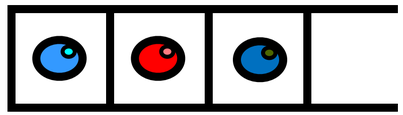
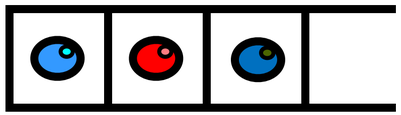
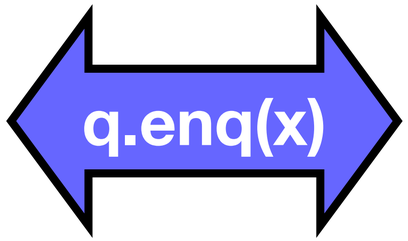
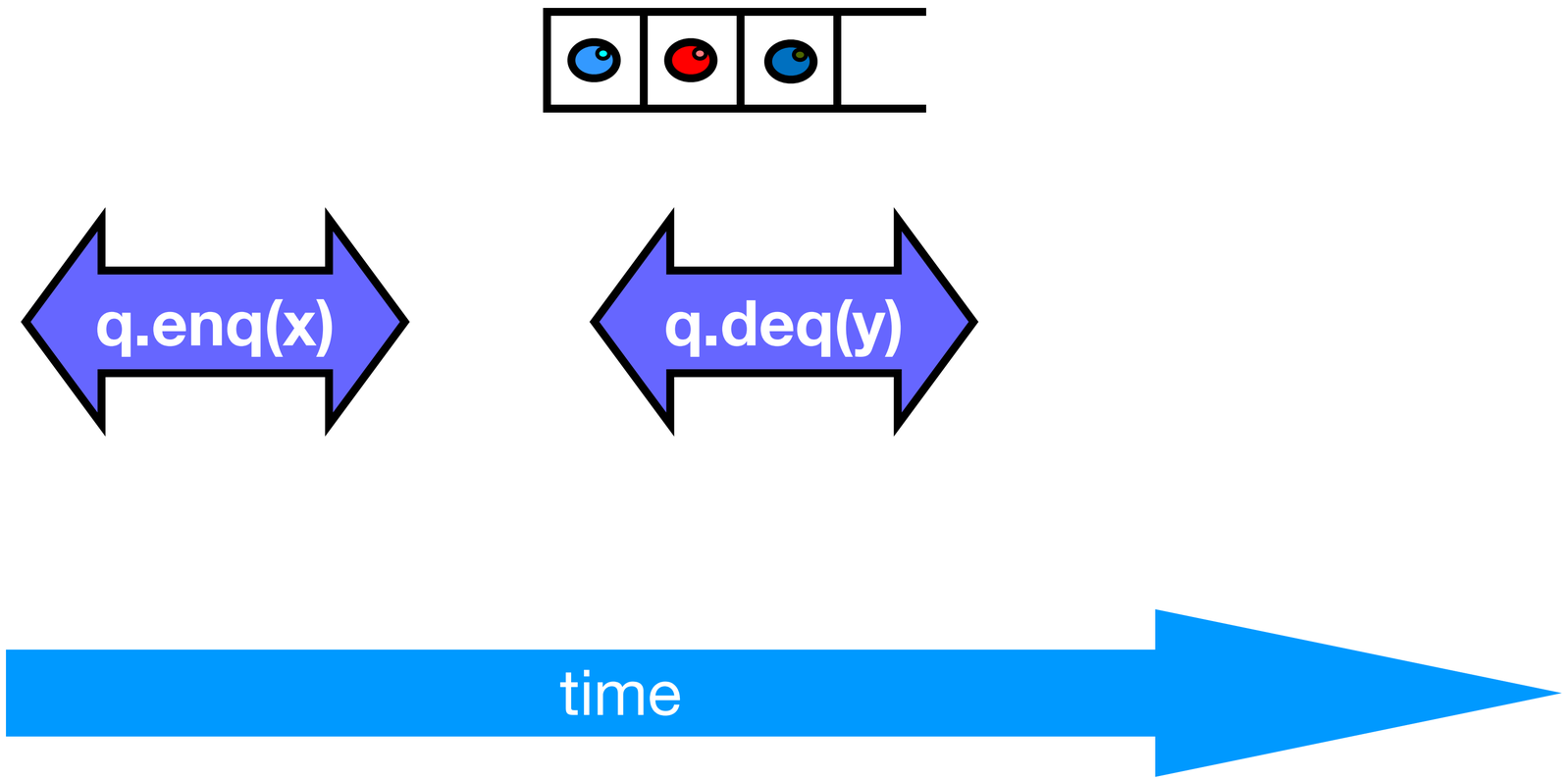


Example

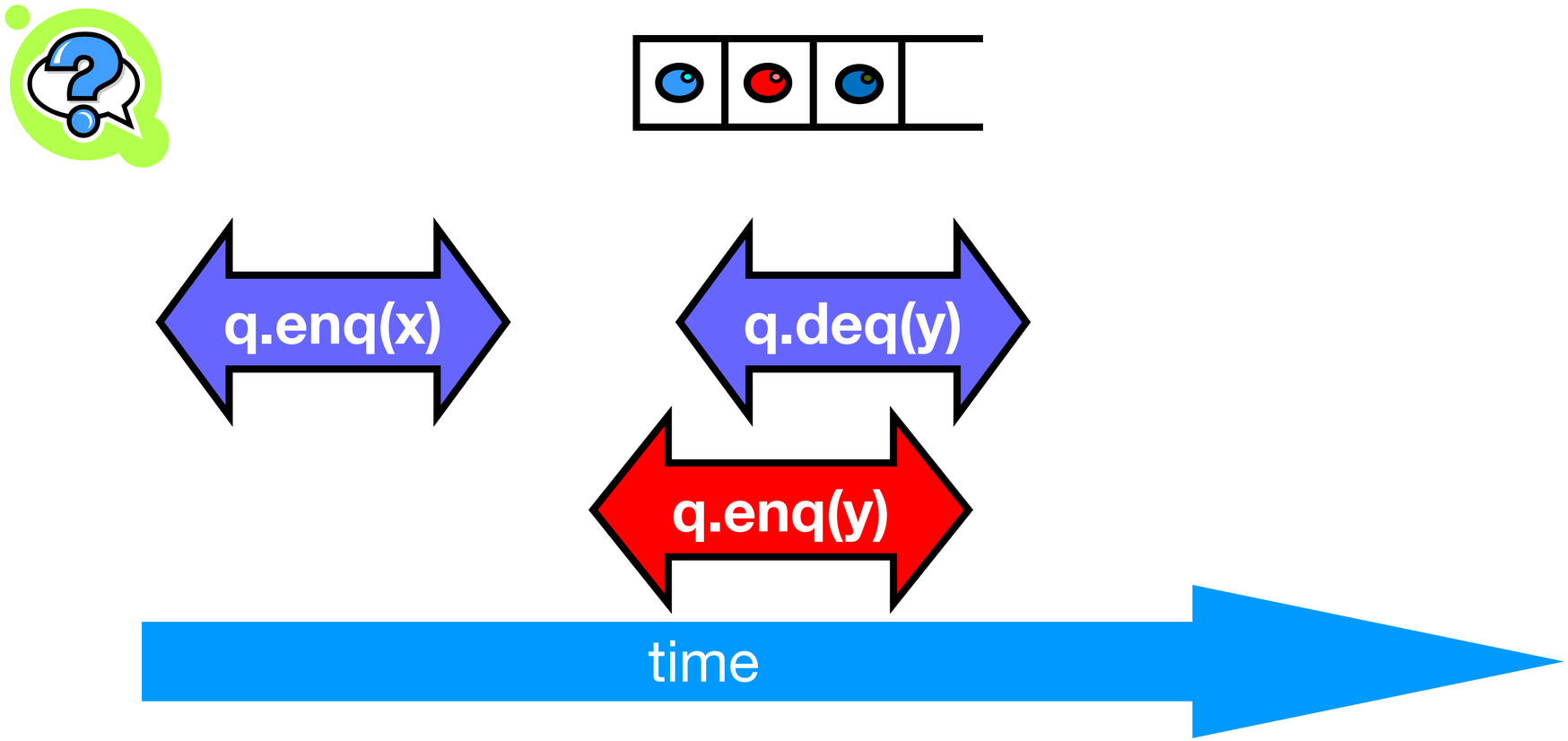




Example



Example



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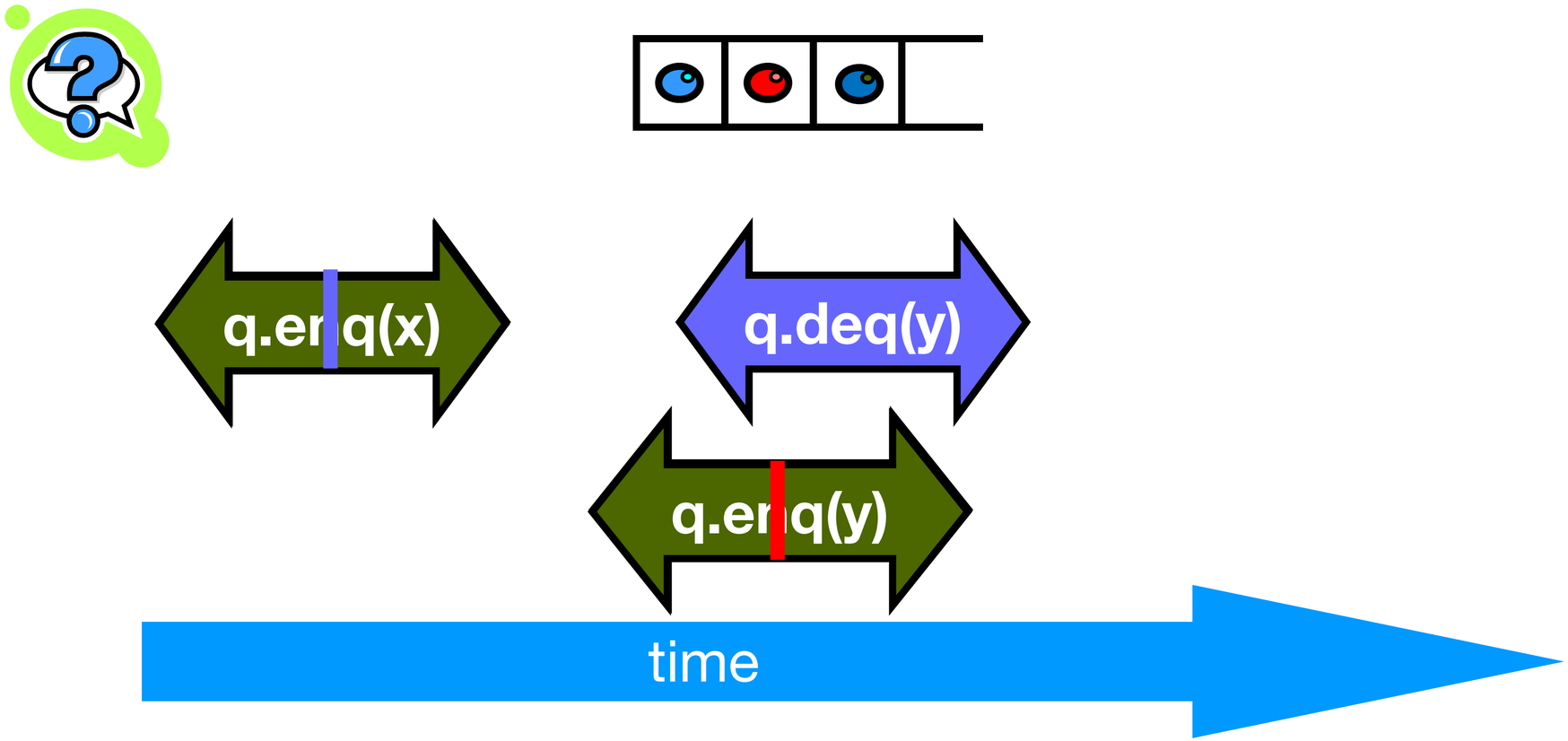
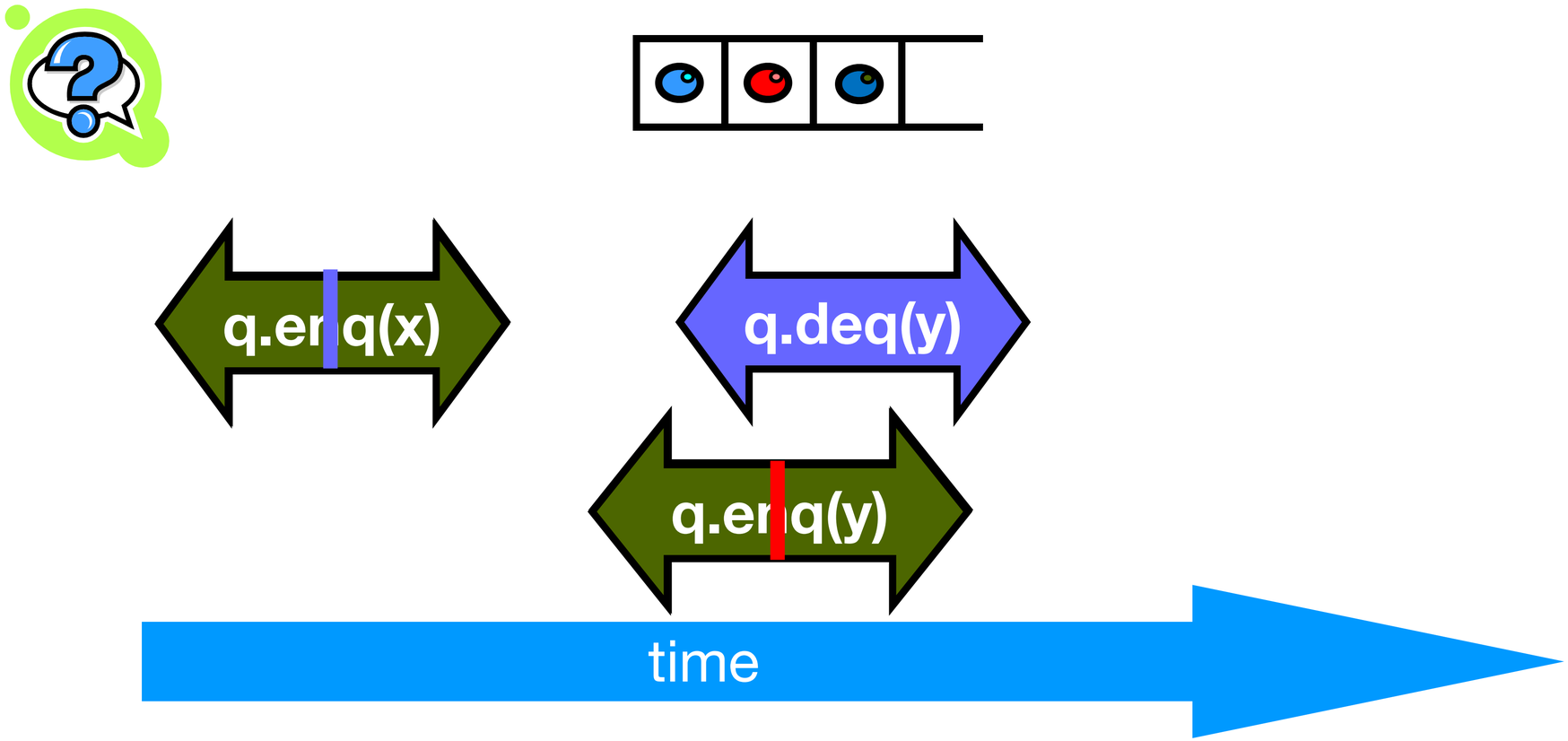
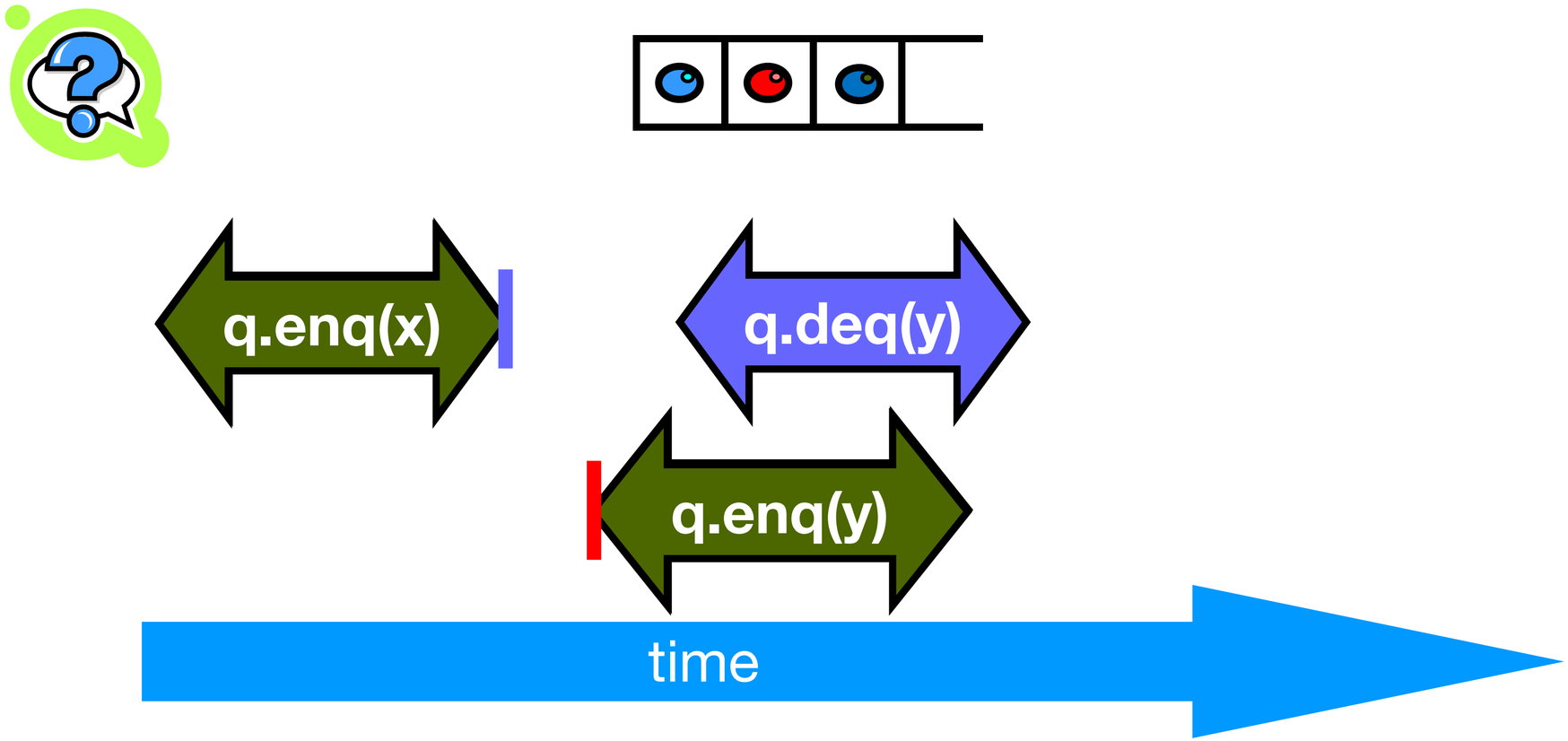


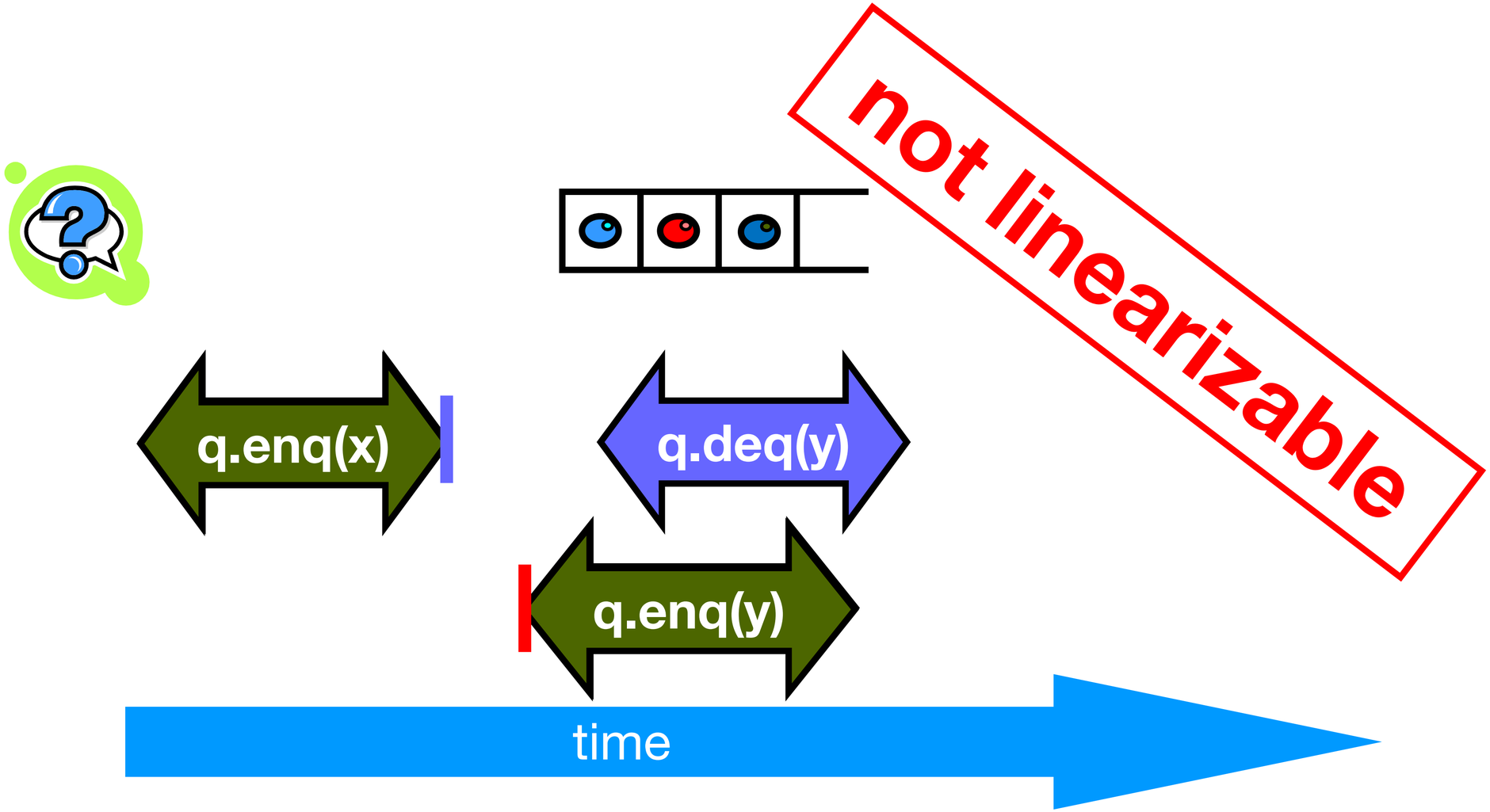
Example



Example



Example



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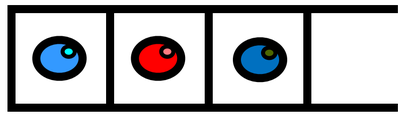

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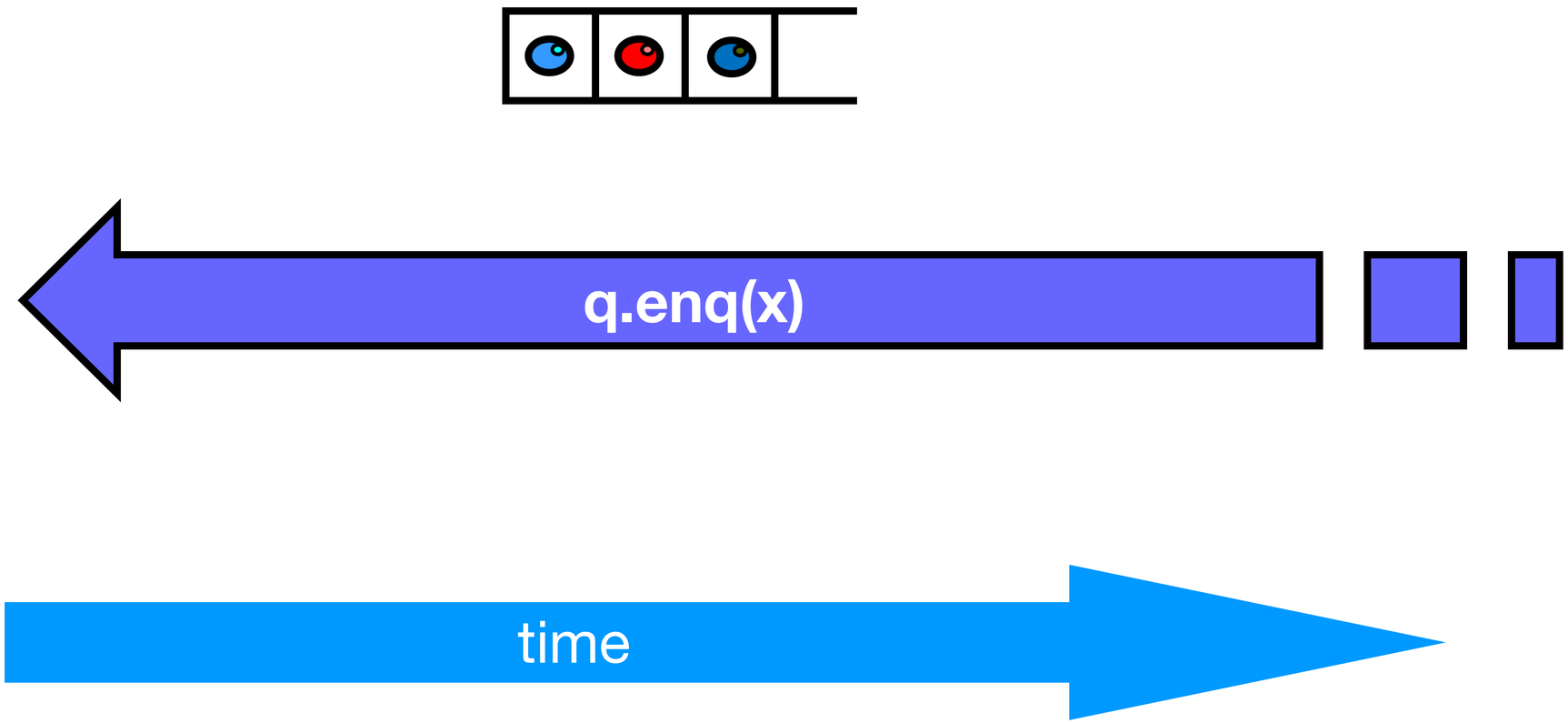
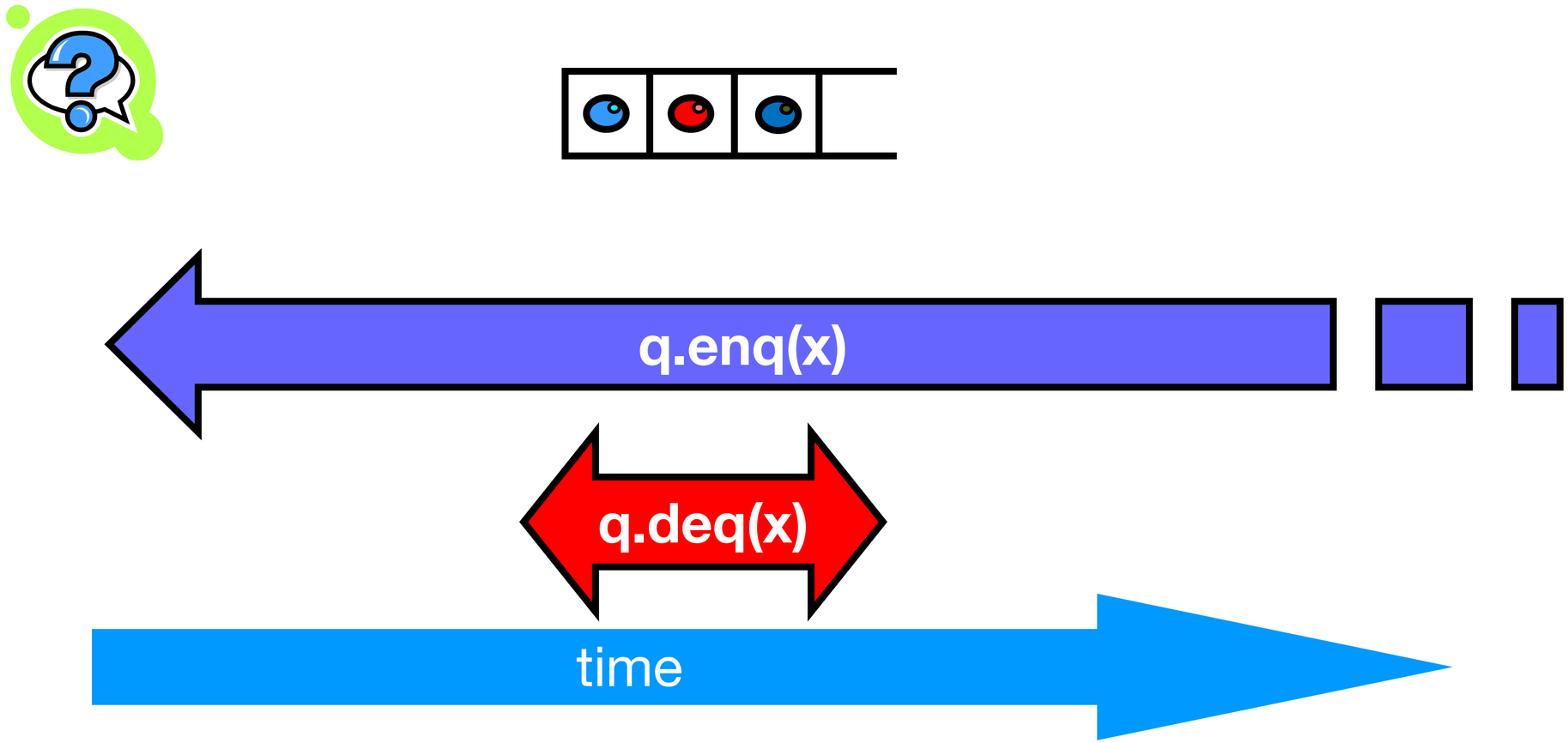
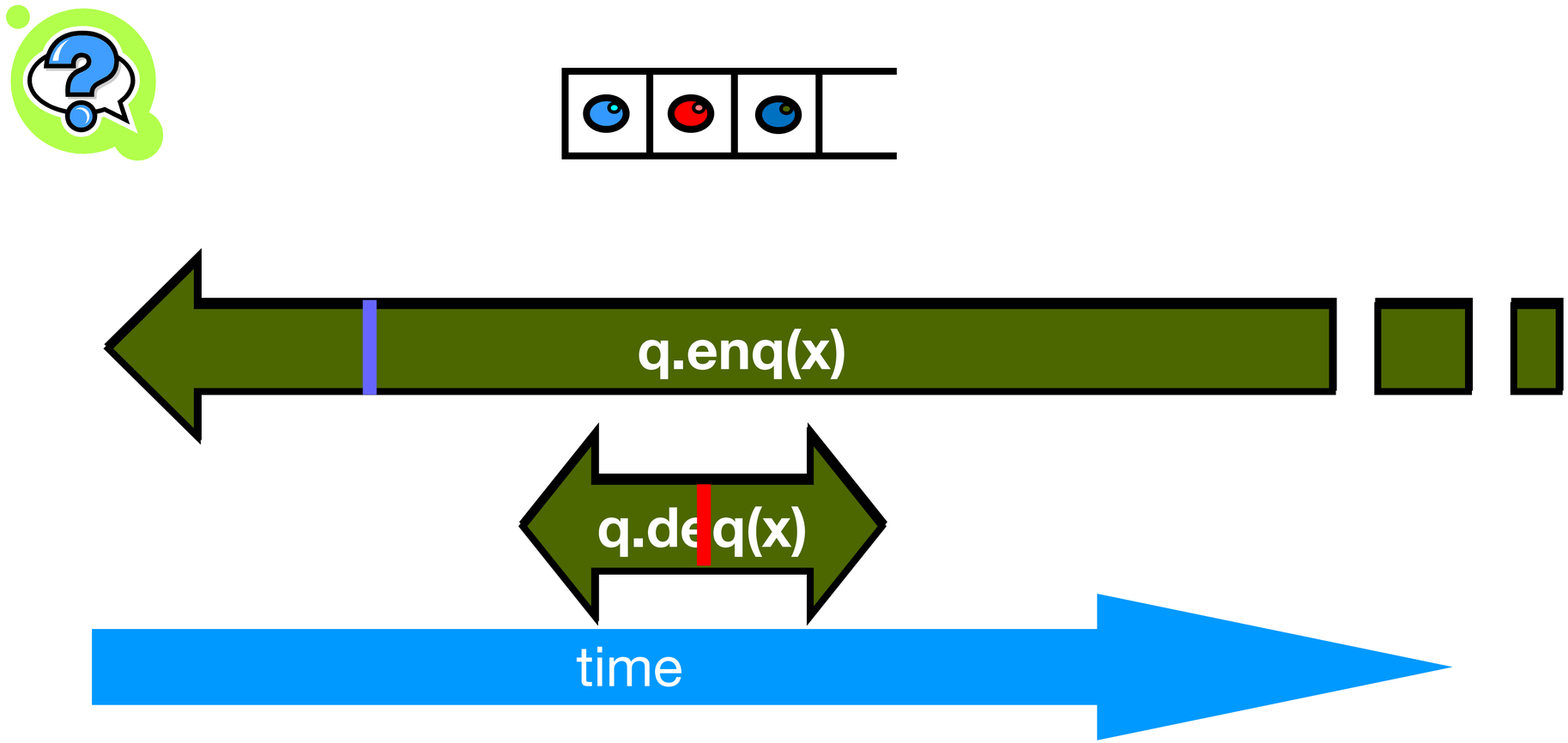


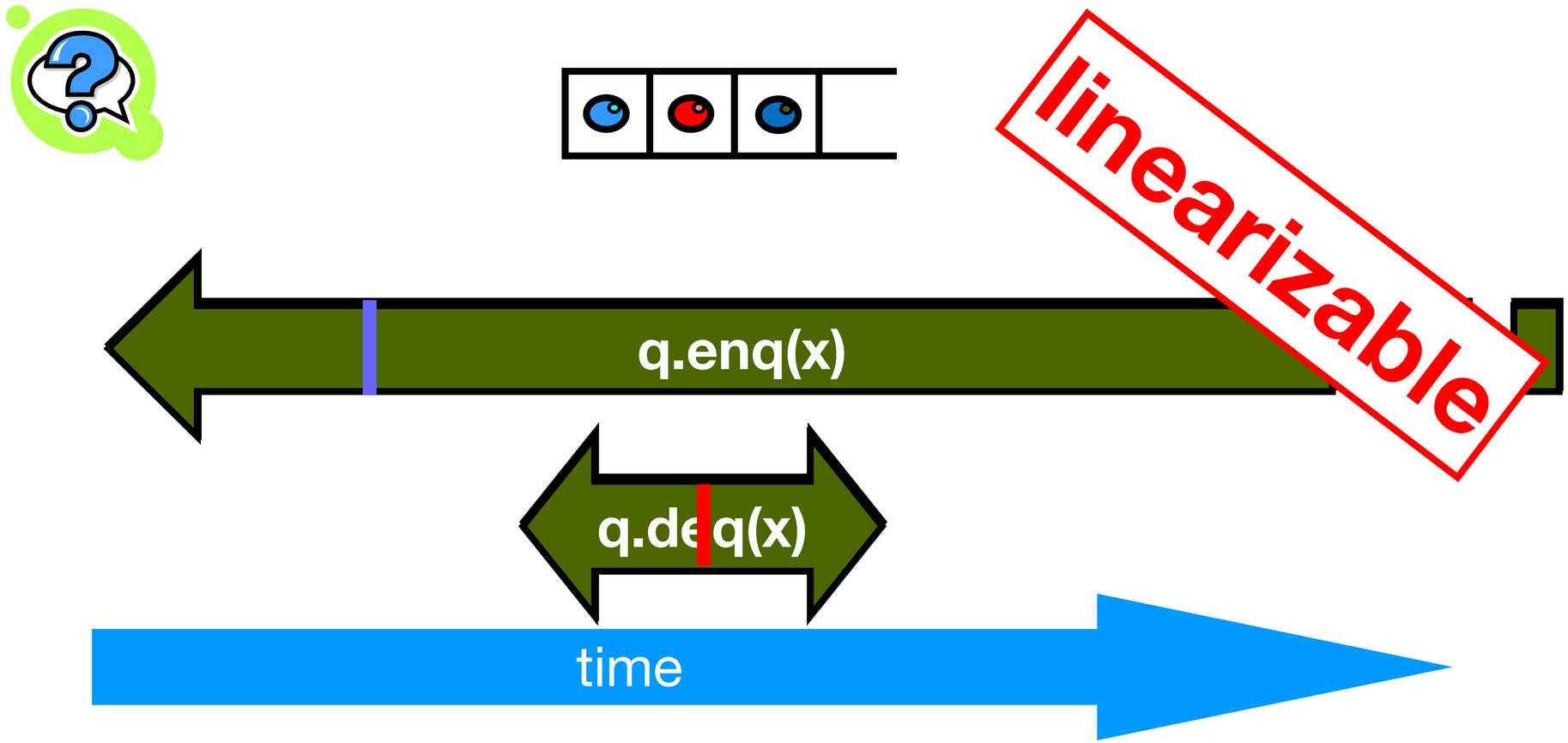
Example



Example



Example



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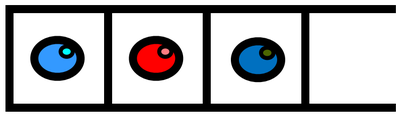

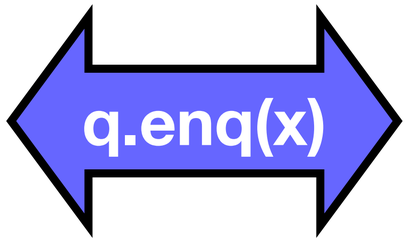

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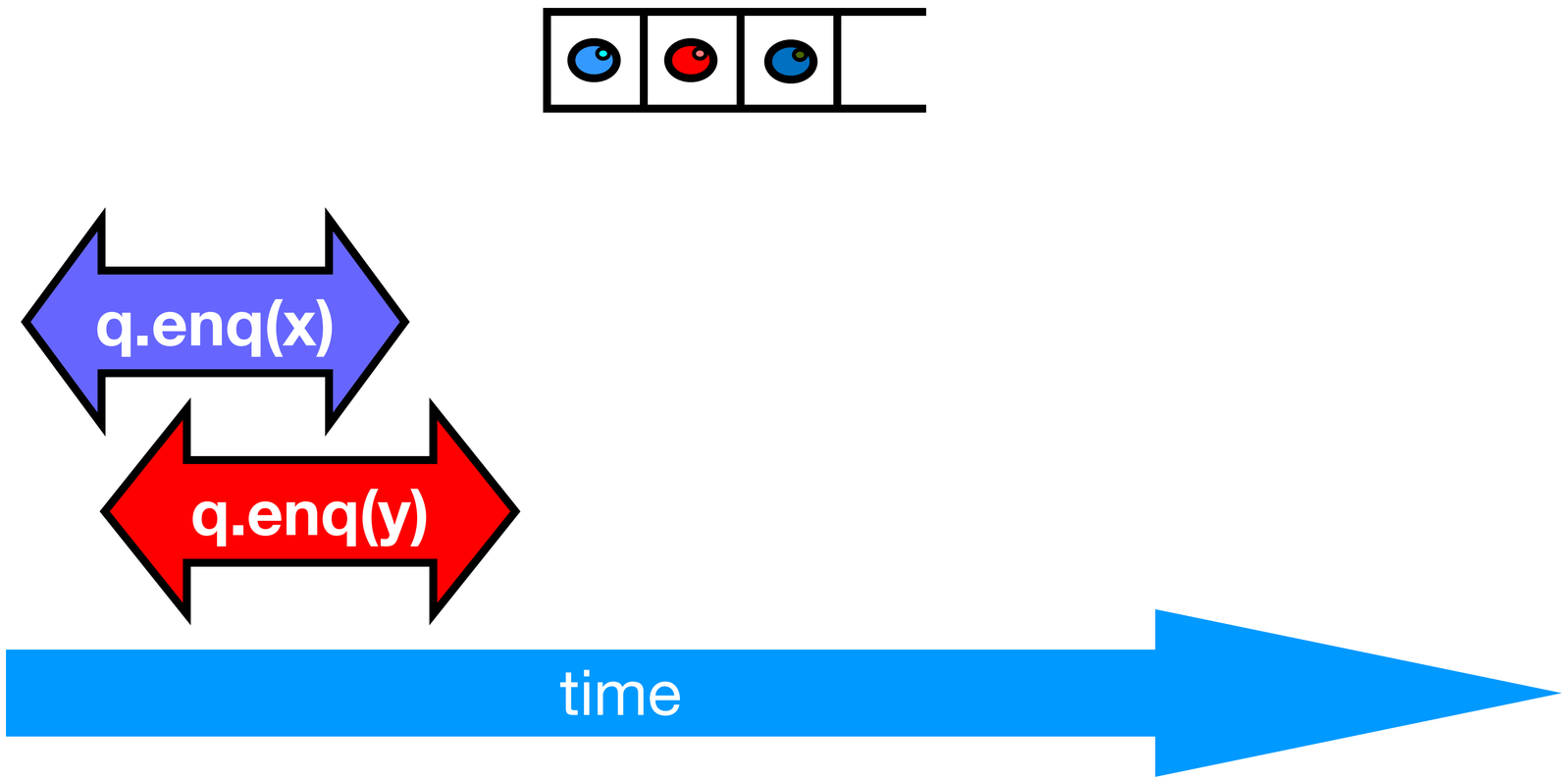

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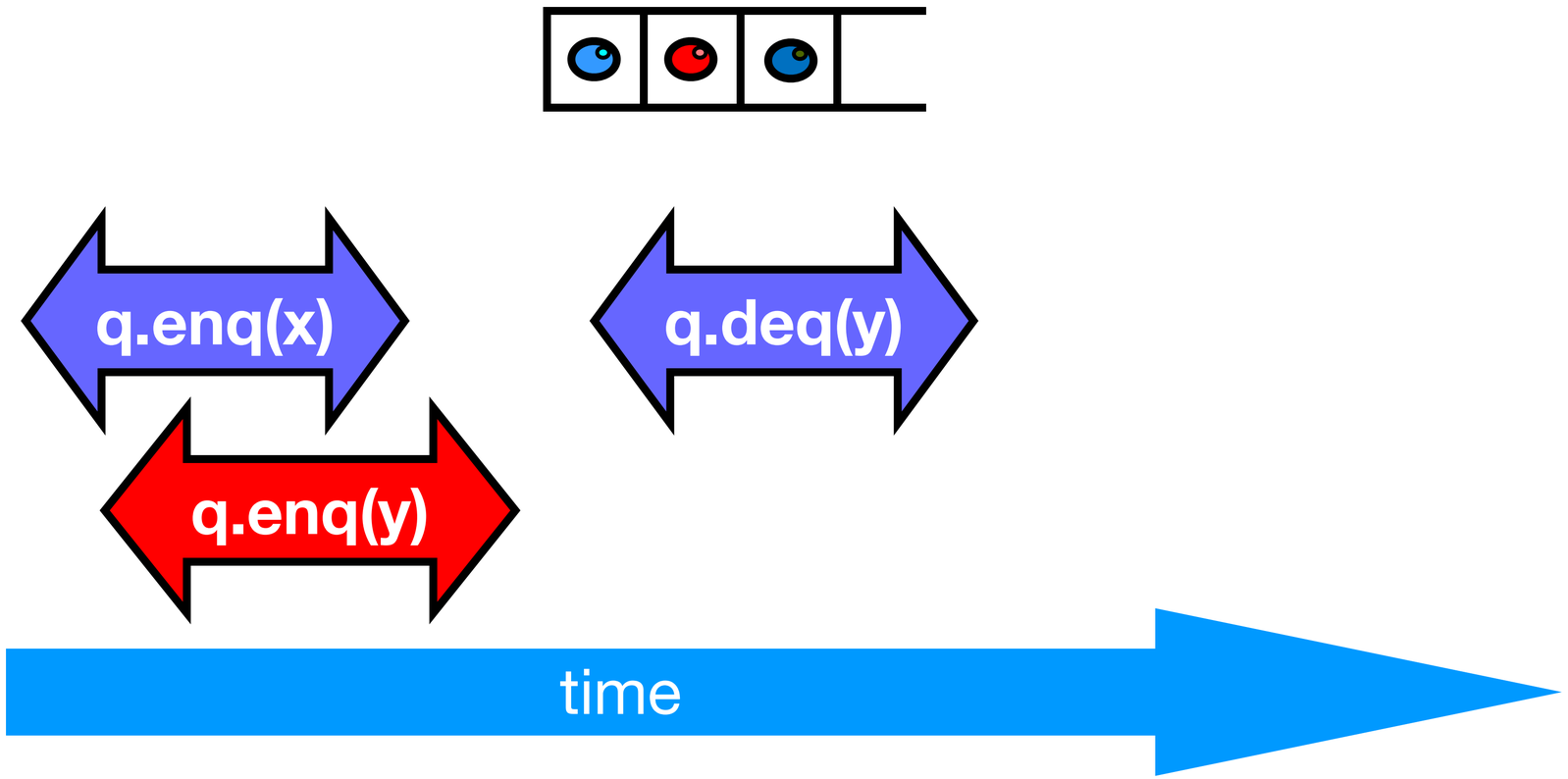

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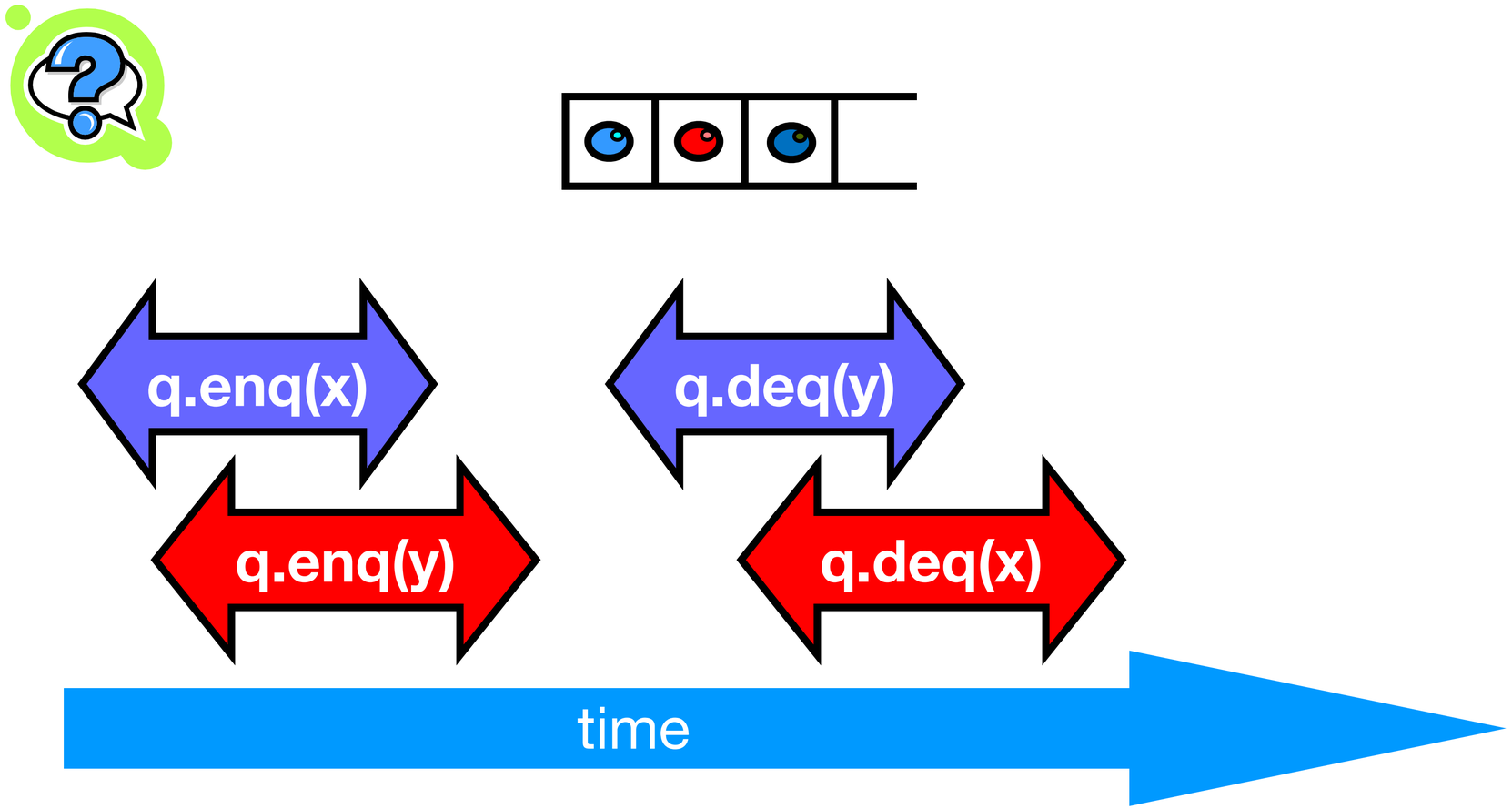

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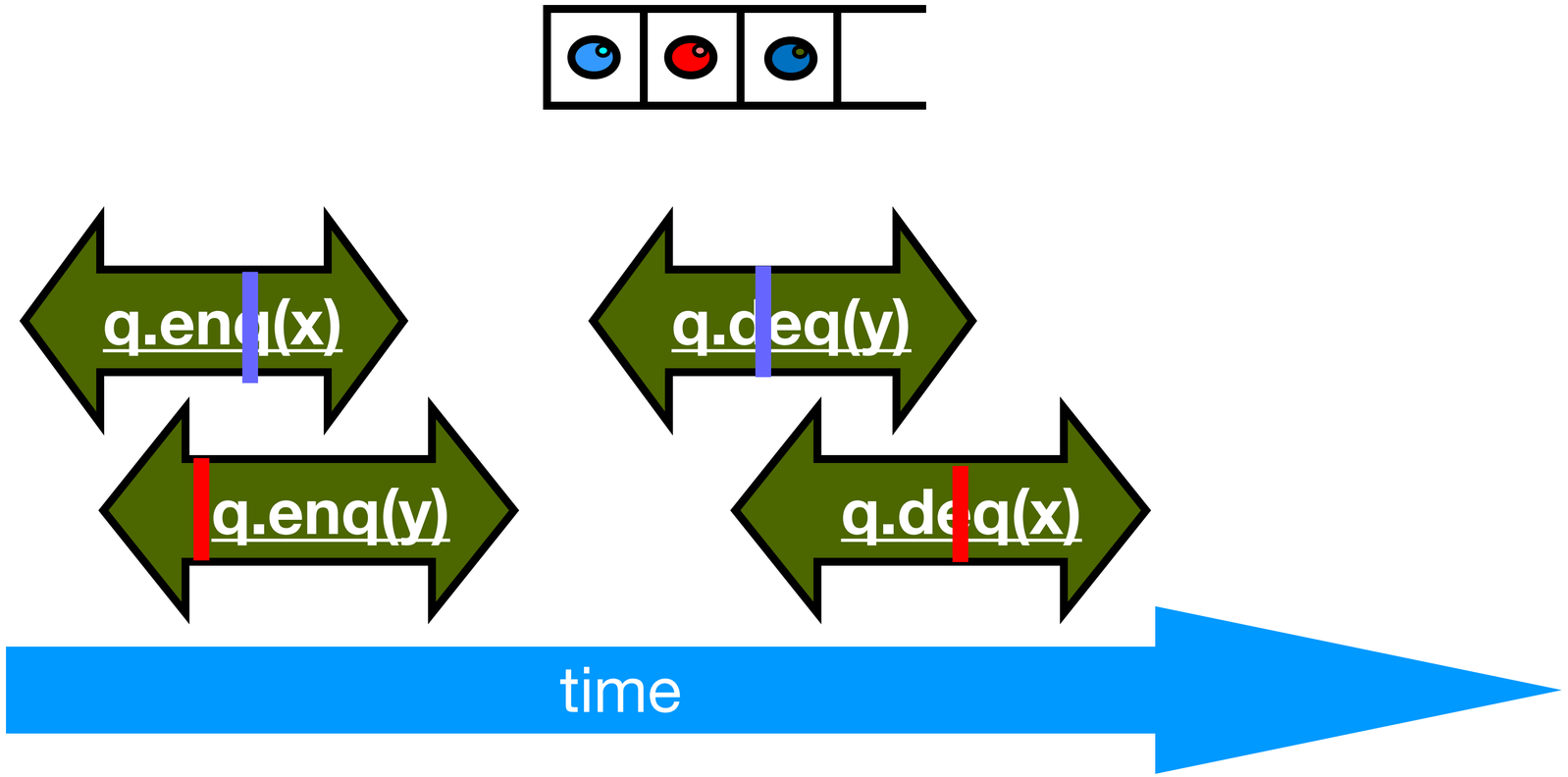

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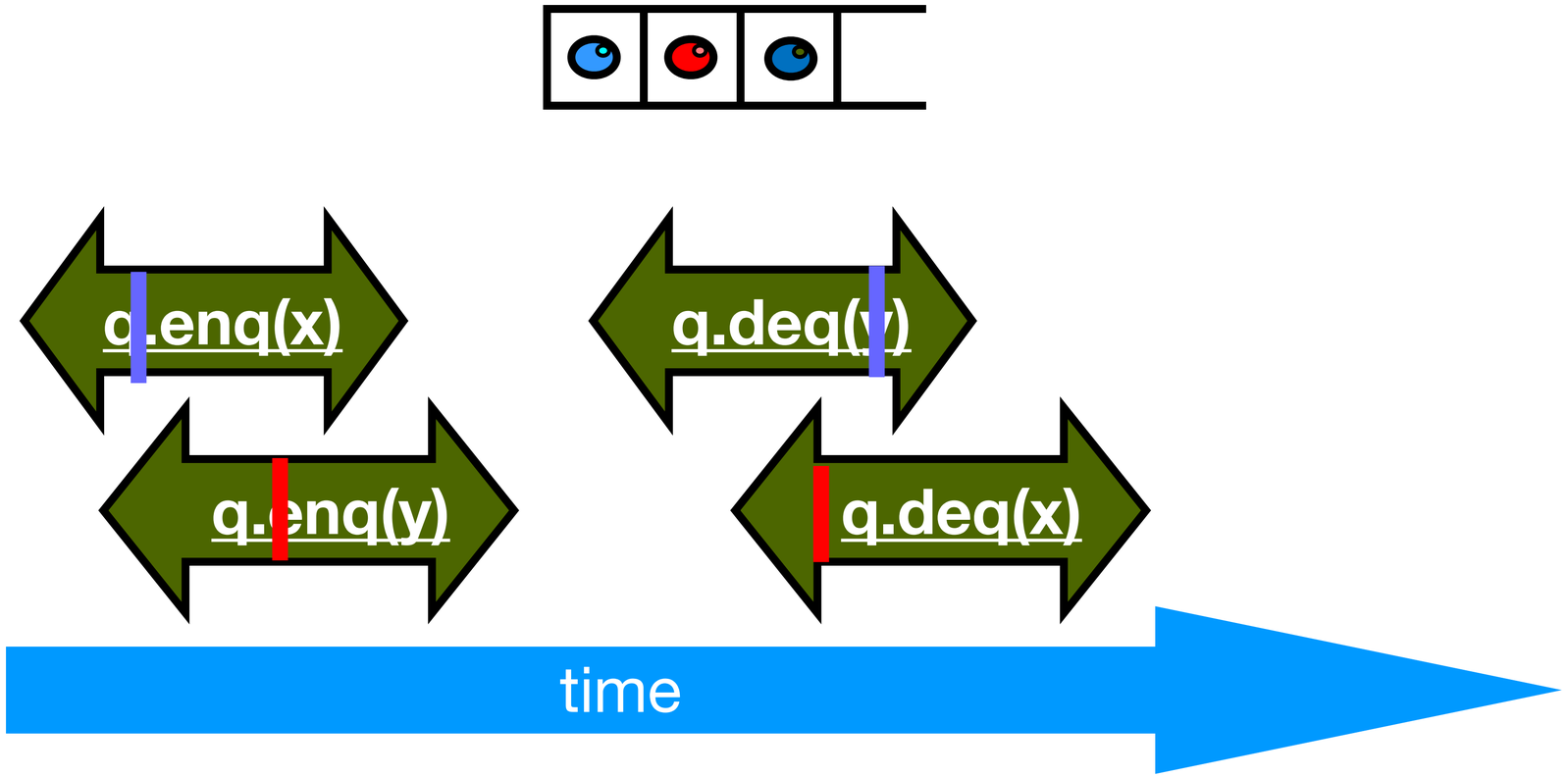

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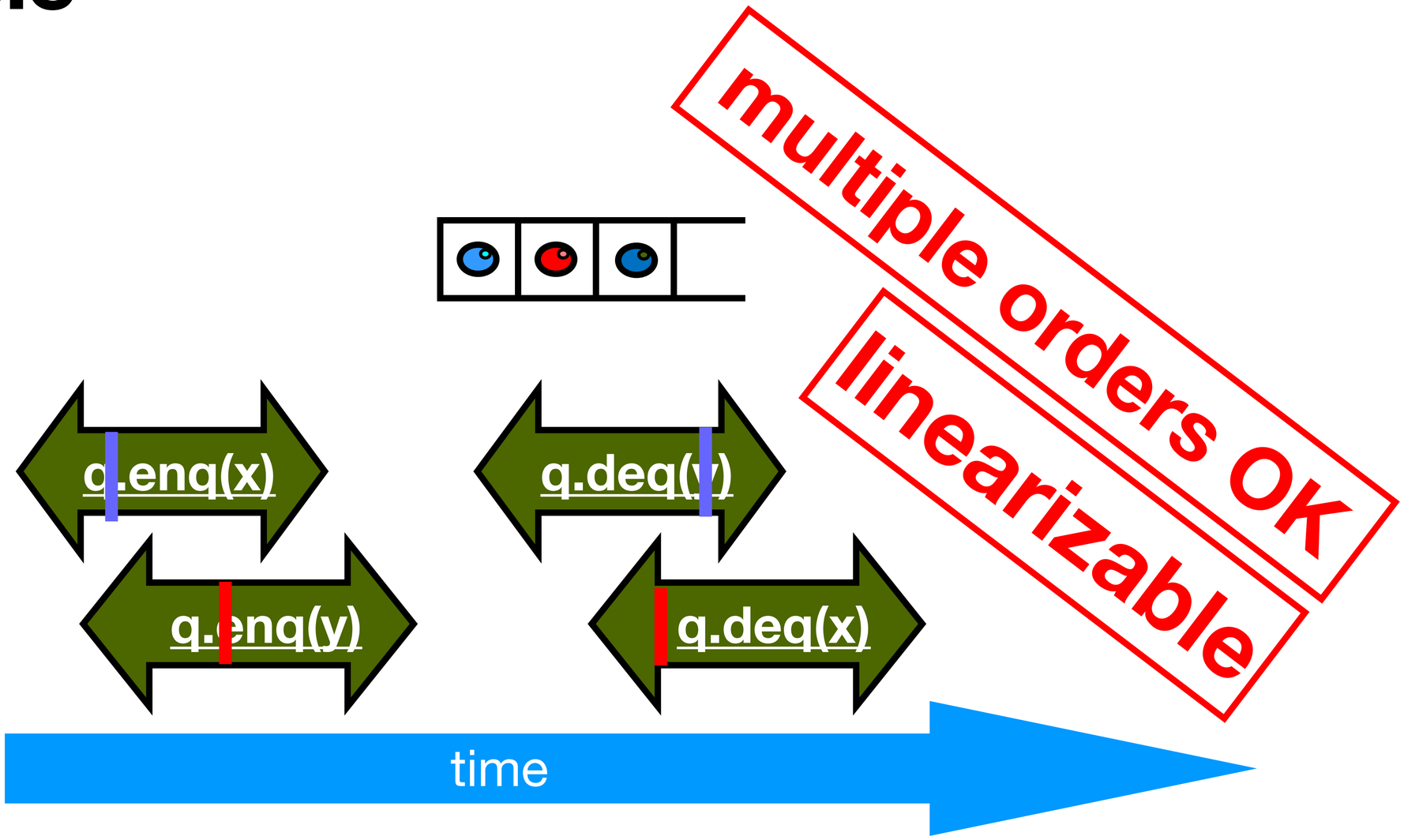
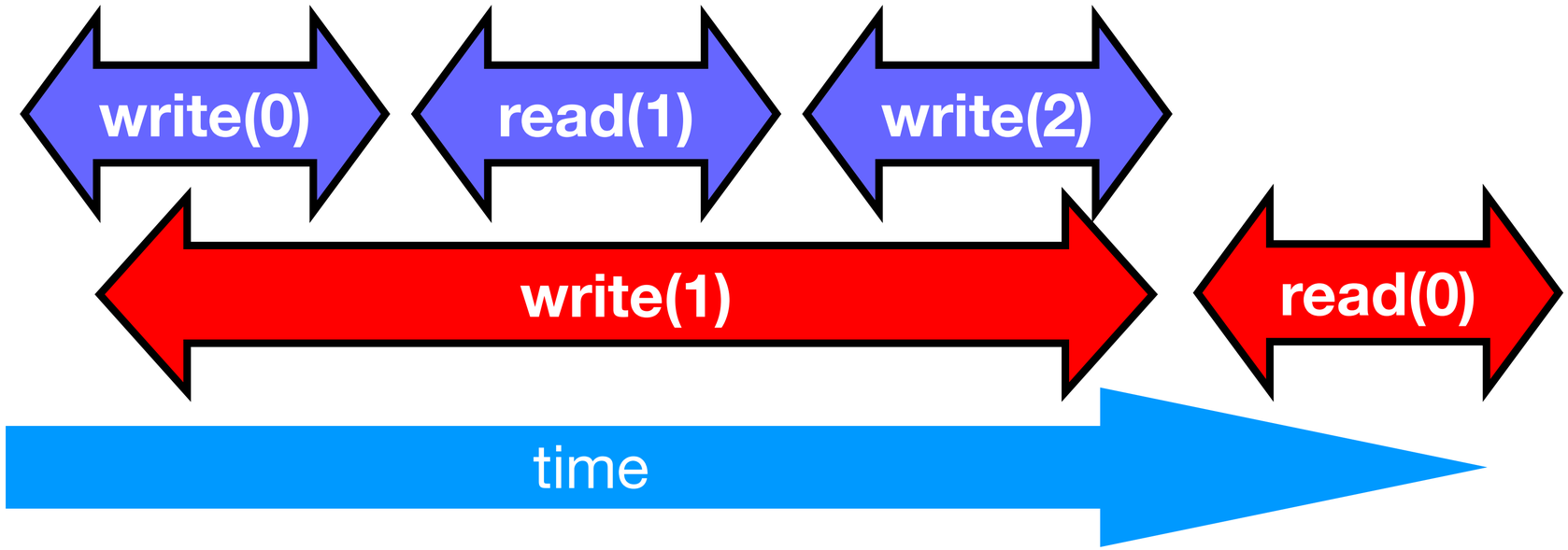
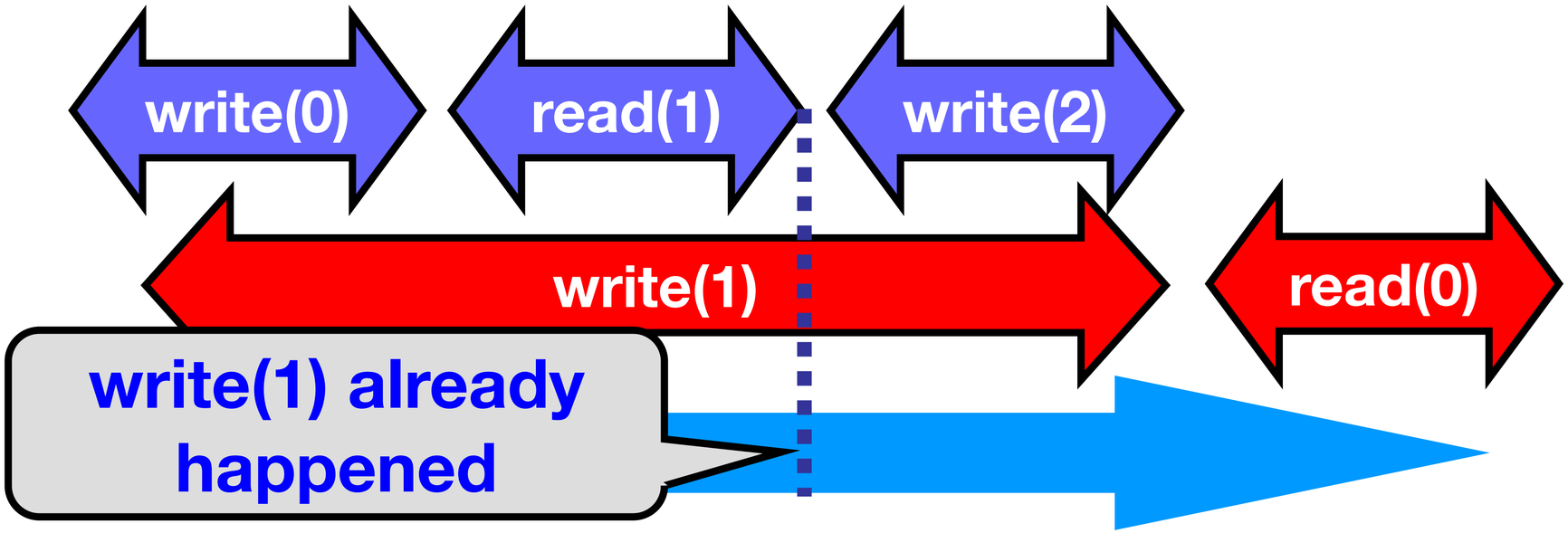

multiple orders OK
linearizable
q.enq(x)
q.deq(y)
q.enq(y)
q.deq(x)
time

Read/Write Register Example



Read/Write Register Example



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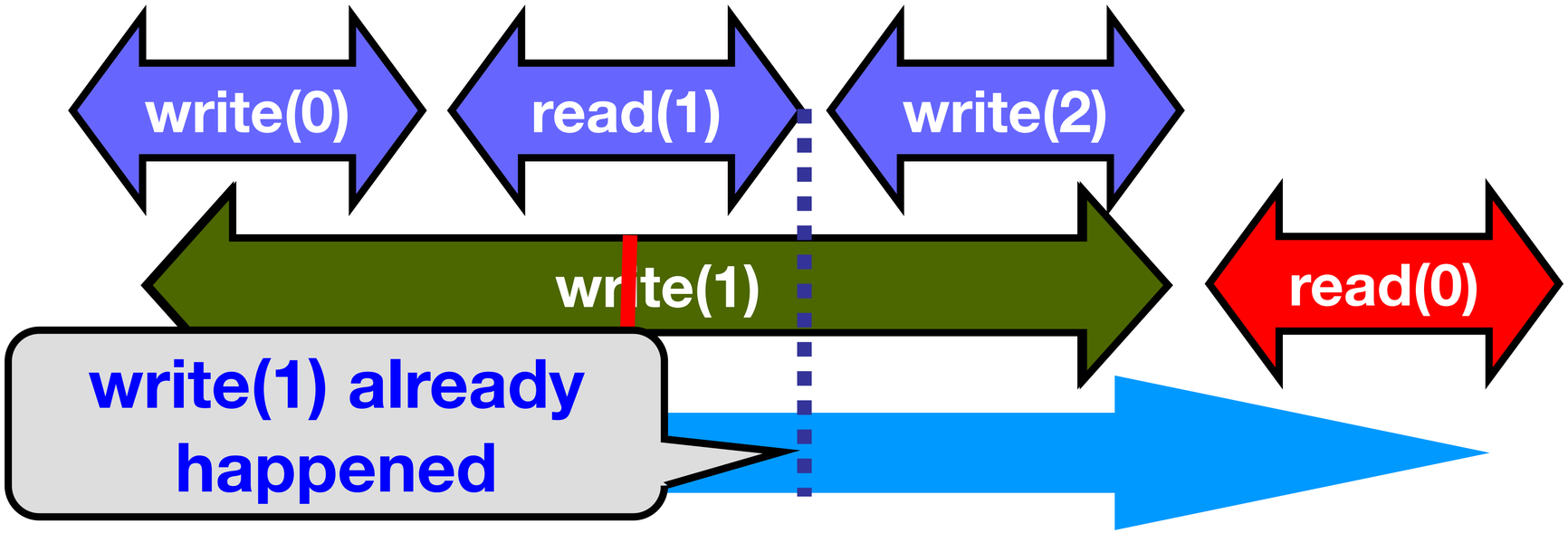

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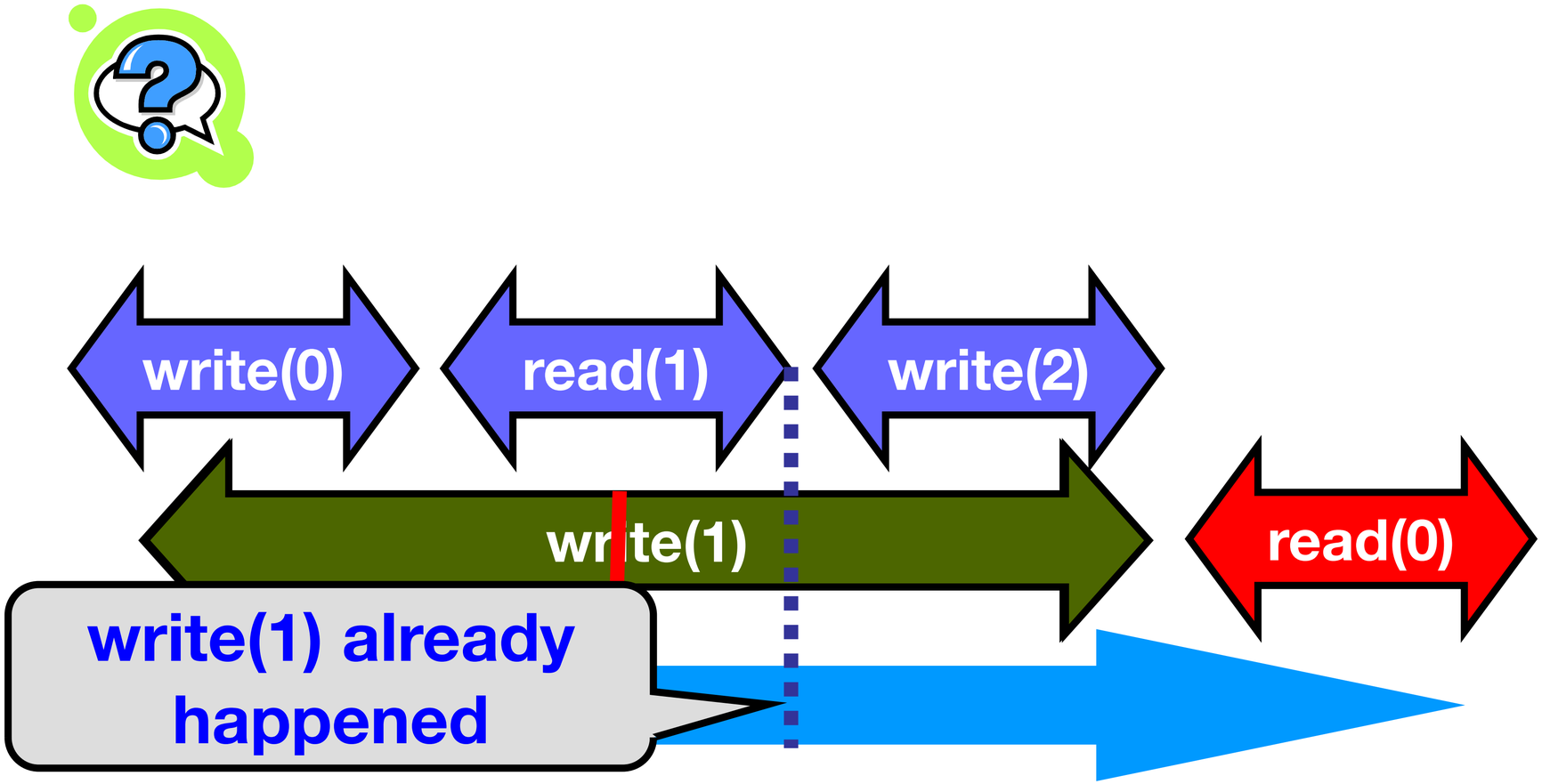

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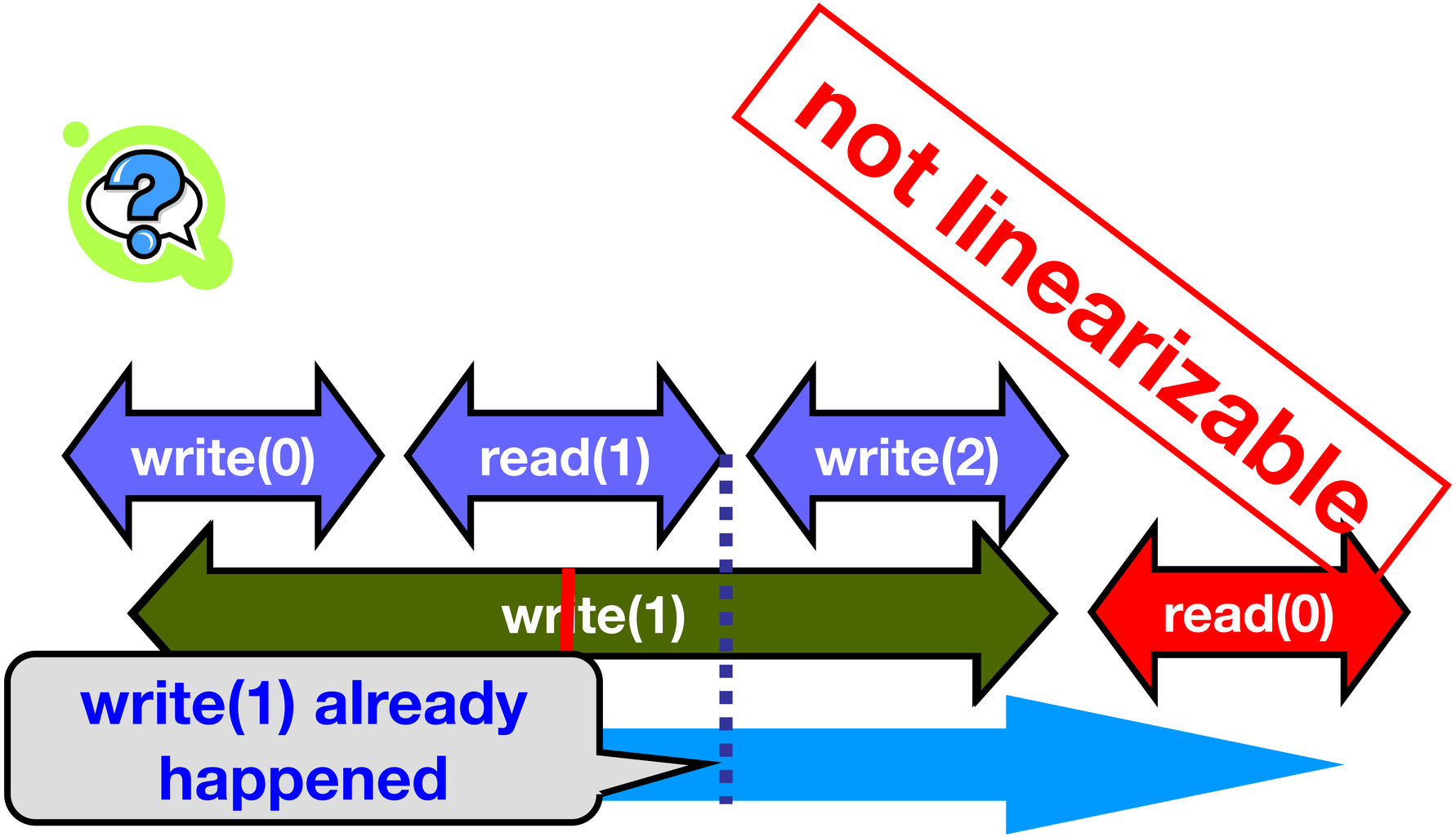

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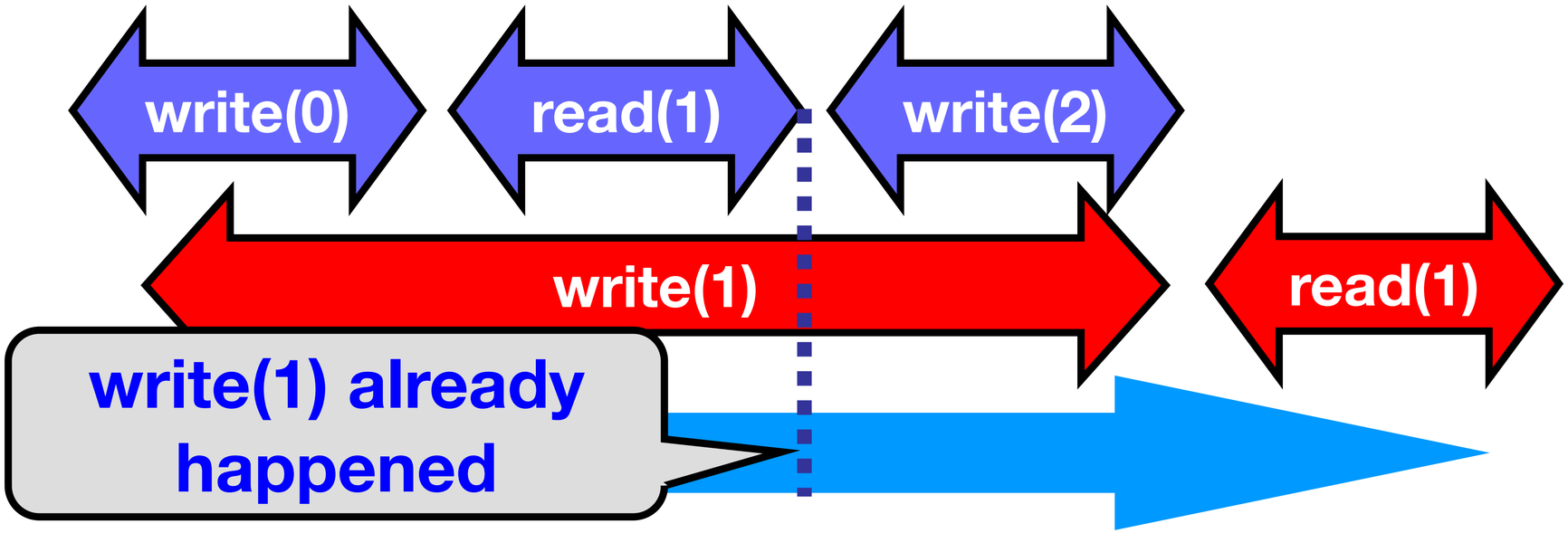

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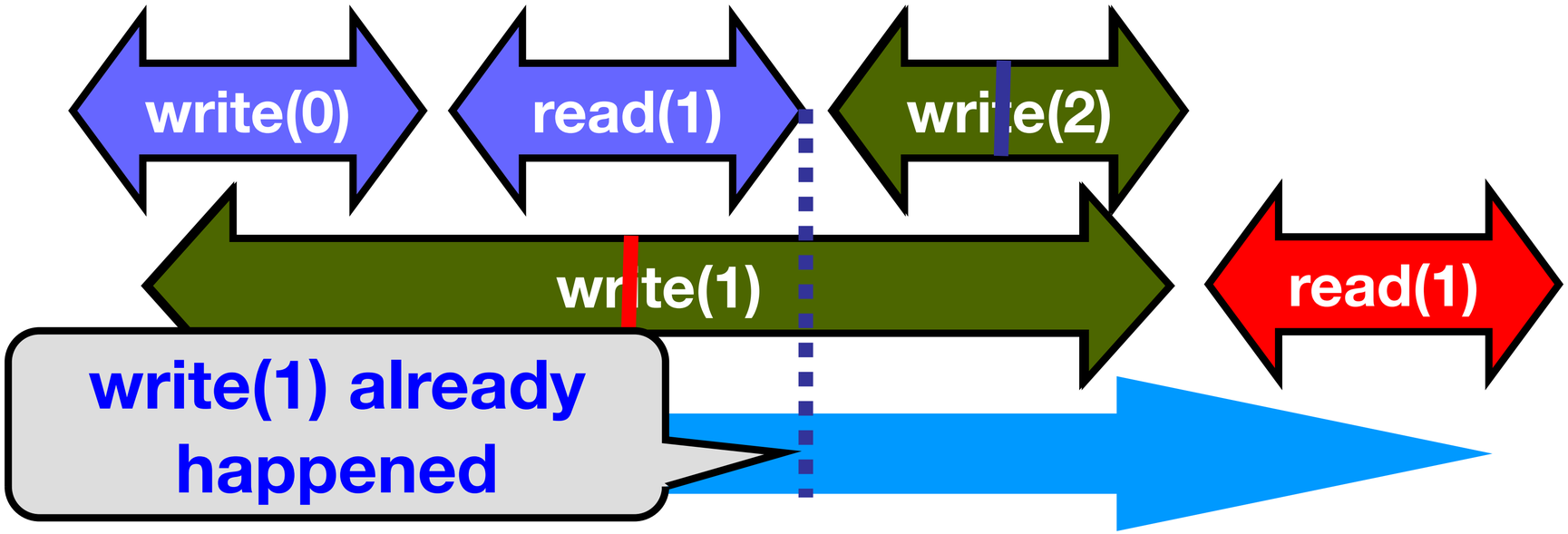

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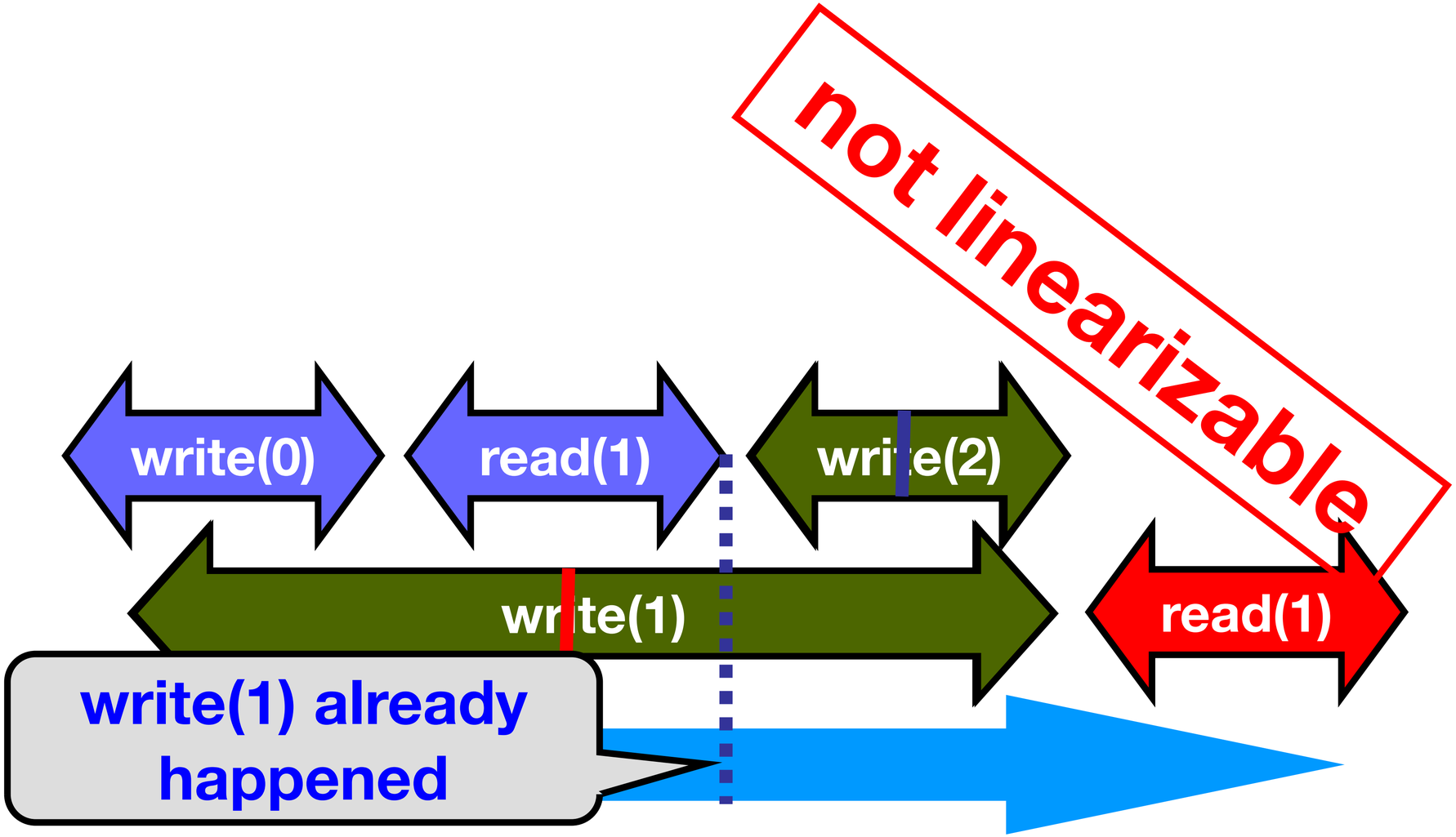

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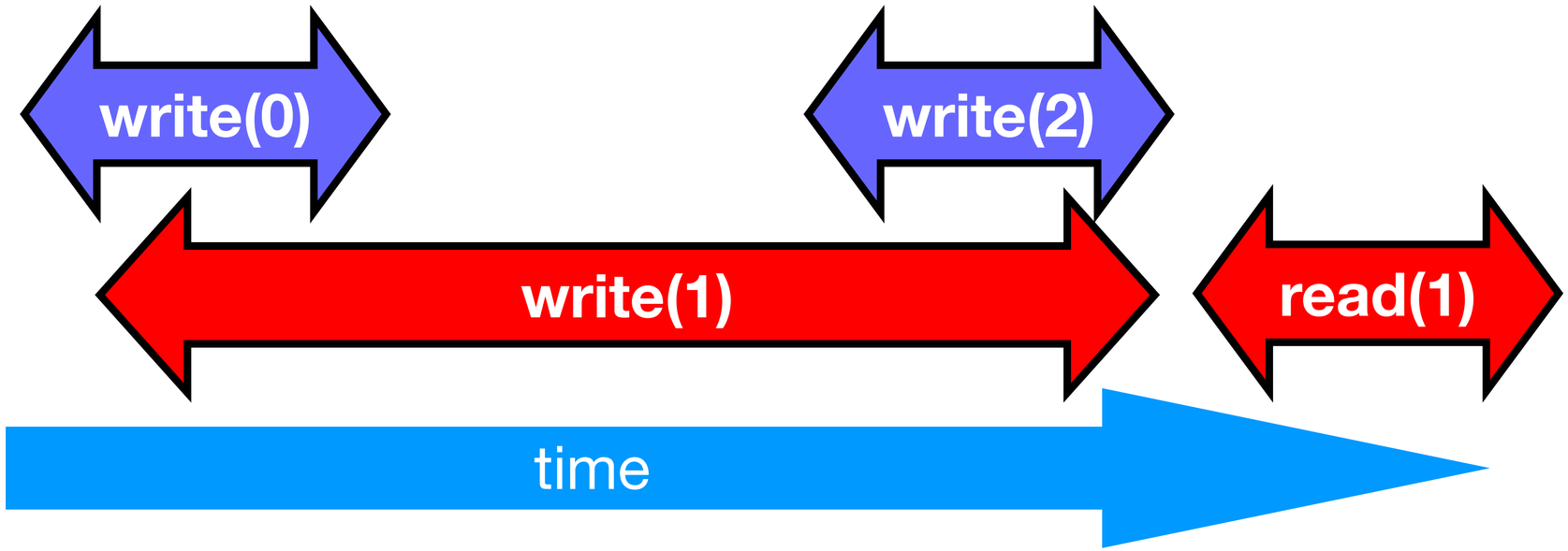

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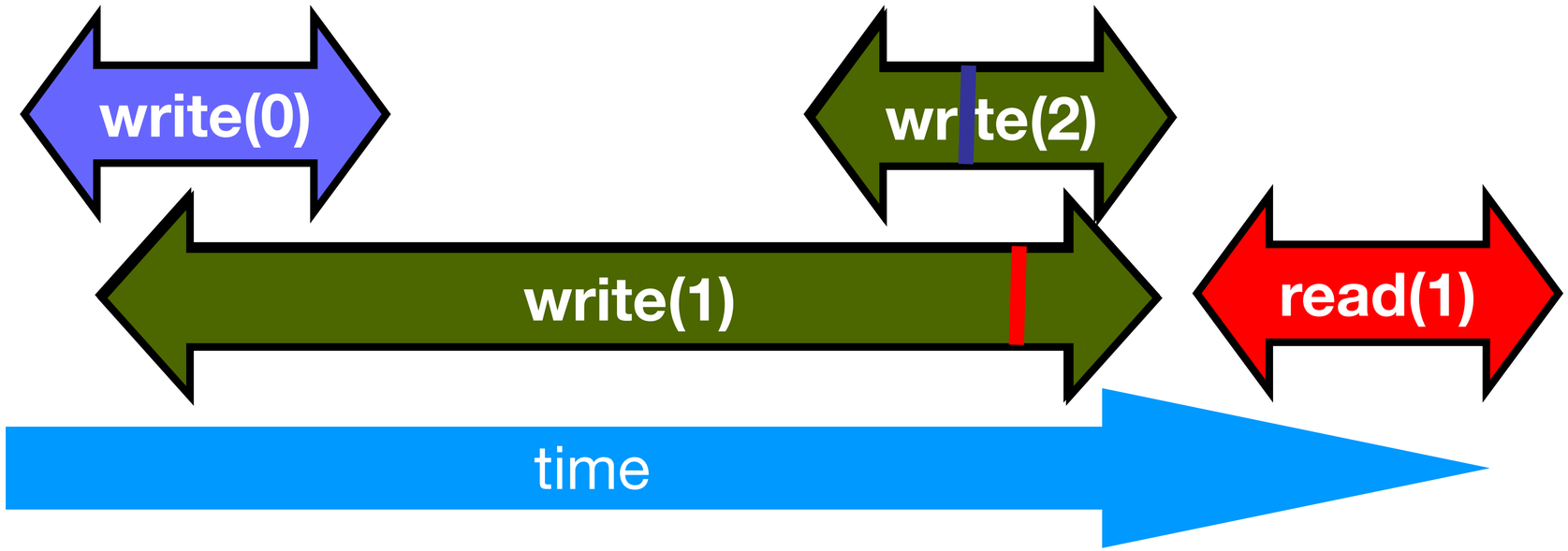

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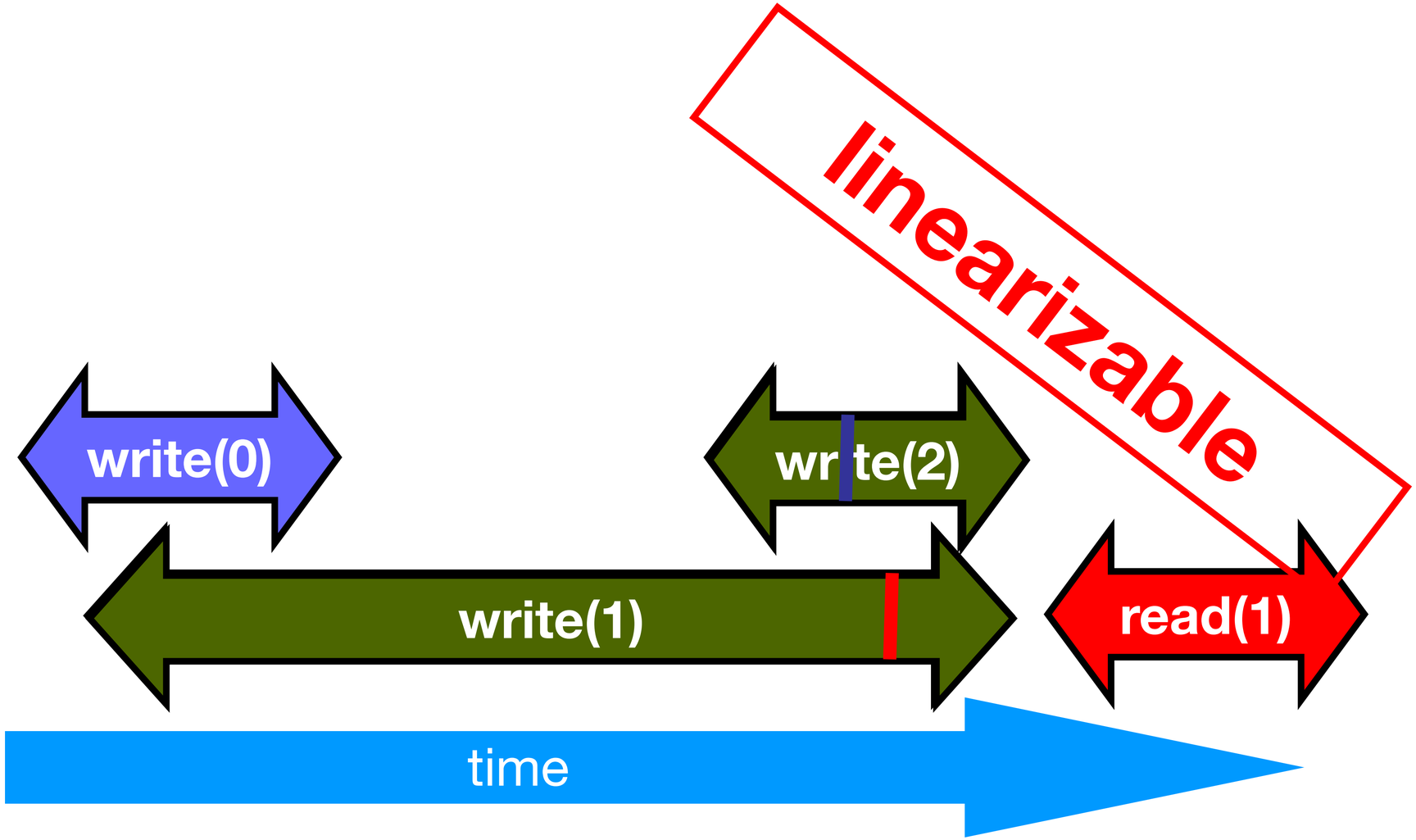

Talking About Executions

- Why?
  - Can’t we specify the linearization point of each operation without describing an execution?
- Not Always
  - In some cases, linearization point ***depends on the execution***

Formal Model of Executions

- Define precisely what we mean
  - Ambiguity is bad when intuition is weak
- Allow reasoning
  - Formal
  - But mostly informal
    - In the long run, actually, more important

Split Method Calls into Two Events

- Invocation
  - method name & args
  - **`q.enq(x)`**
- Response
  - result or exception
  - **`q.enq(x)`** returns **`void`**
  - **`q.deq()`** returns **`x`**
  - **`q.deq()`** throws **`empty`**

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- Note that I'm following the convention of the book
  - Book uses OO
  - Code in this course uses FP
- Note that we're still reasoning using ***objectivism***
- For the current discussion, distinction doesn't matter
  - **`q.enq(x)`** is read as **`enq q x`** in code
  - Returns **`void`** is read as returns **`()`**
  - Throws **`empty`** is read as raises **`Empty`**

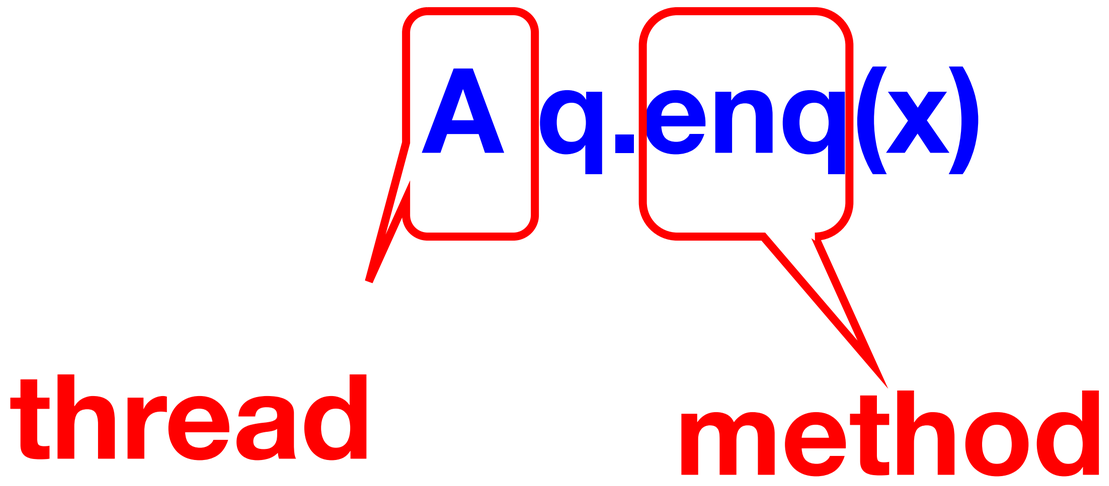
Invocation Notation

A q.enq(x)

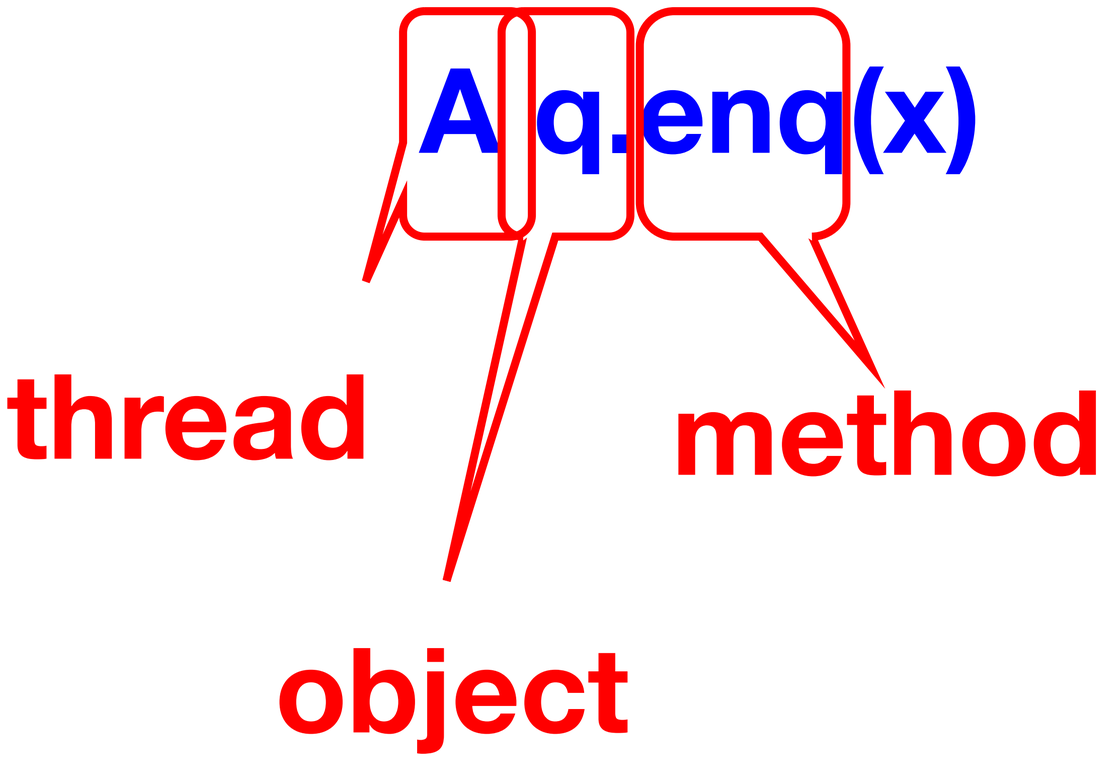
Invocation Notation

thread

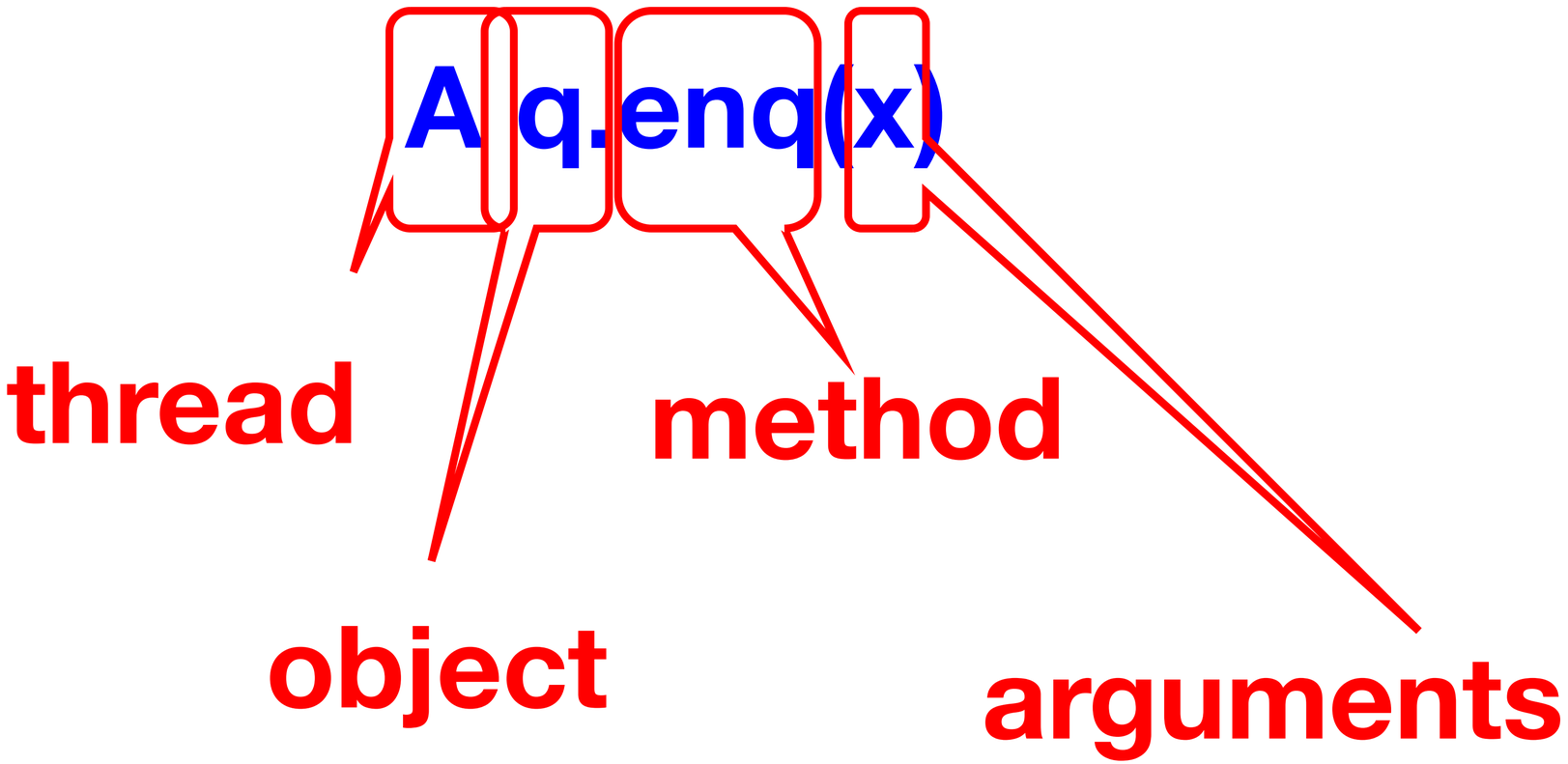
Invocation Notation



Invocation Notation



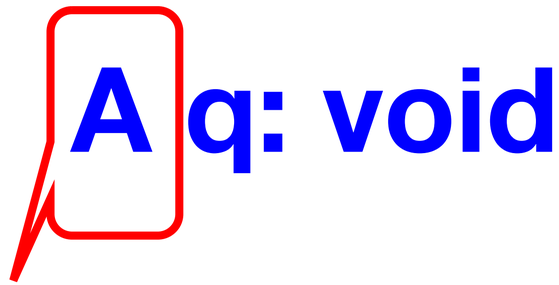
Invocation Notation



Response Notation

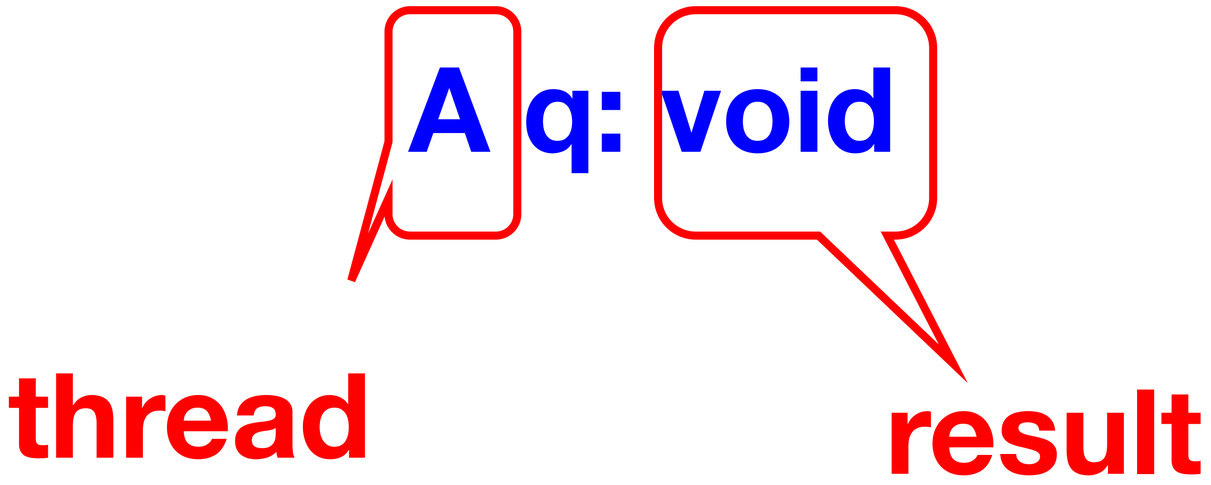
A q: void

Response Notation

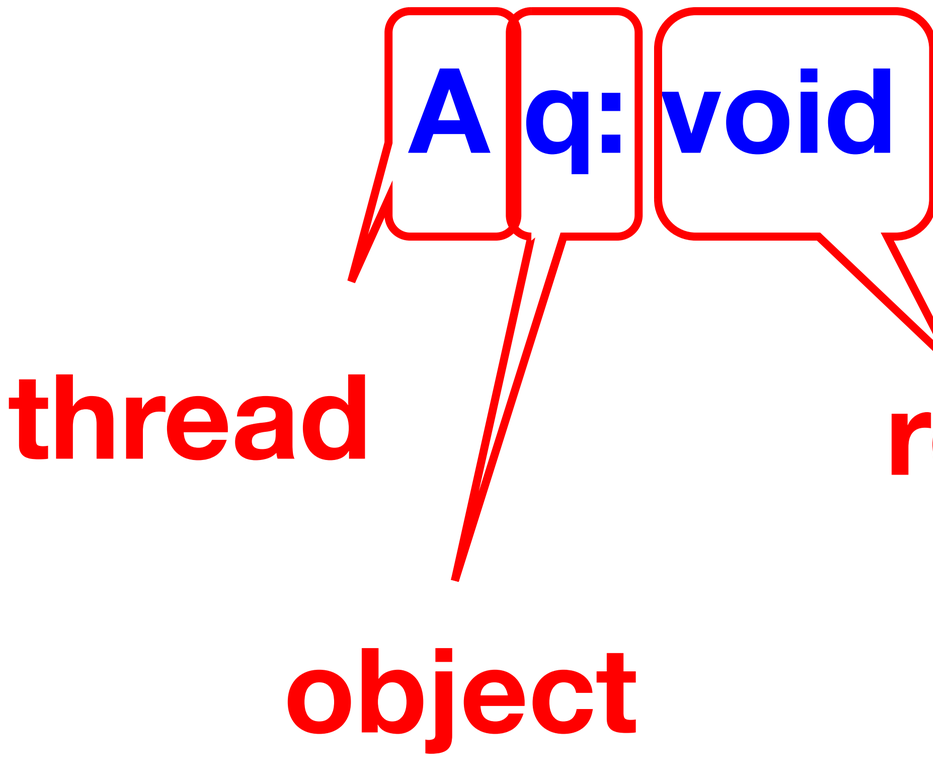


thread

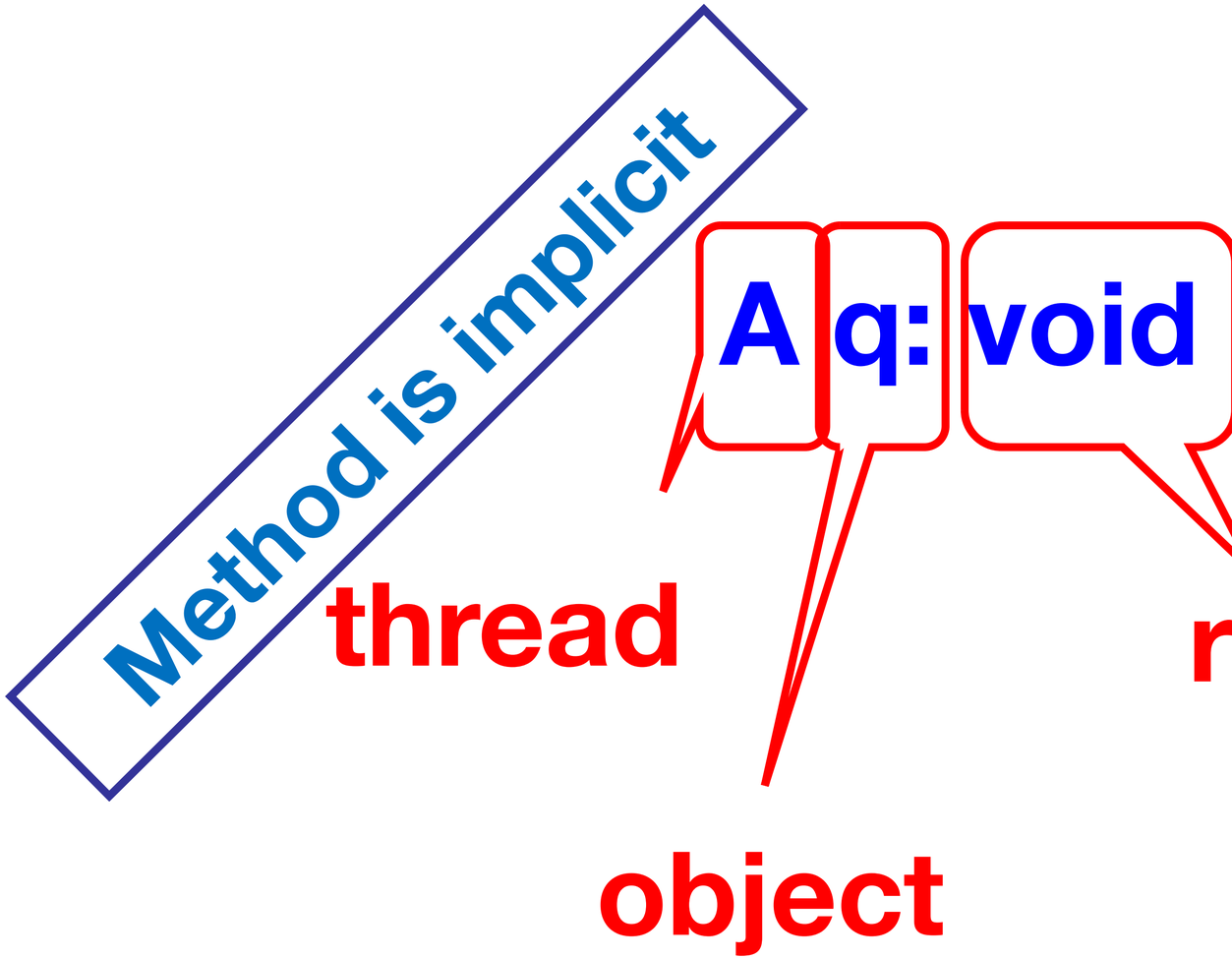
Response Notation



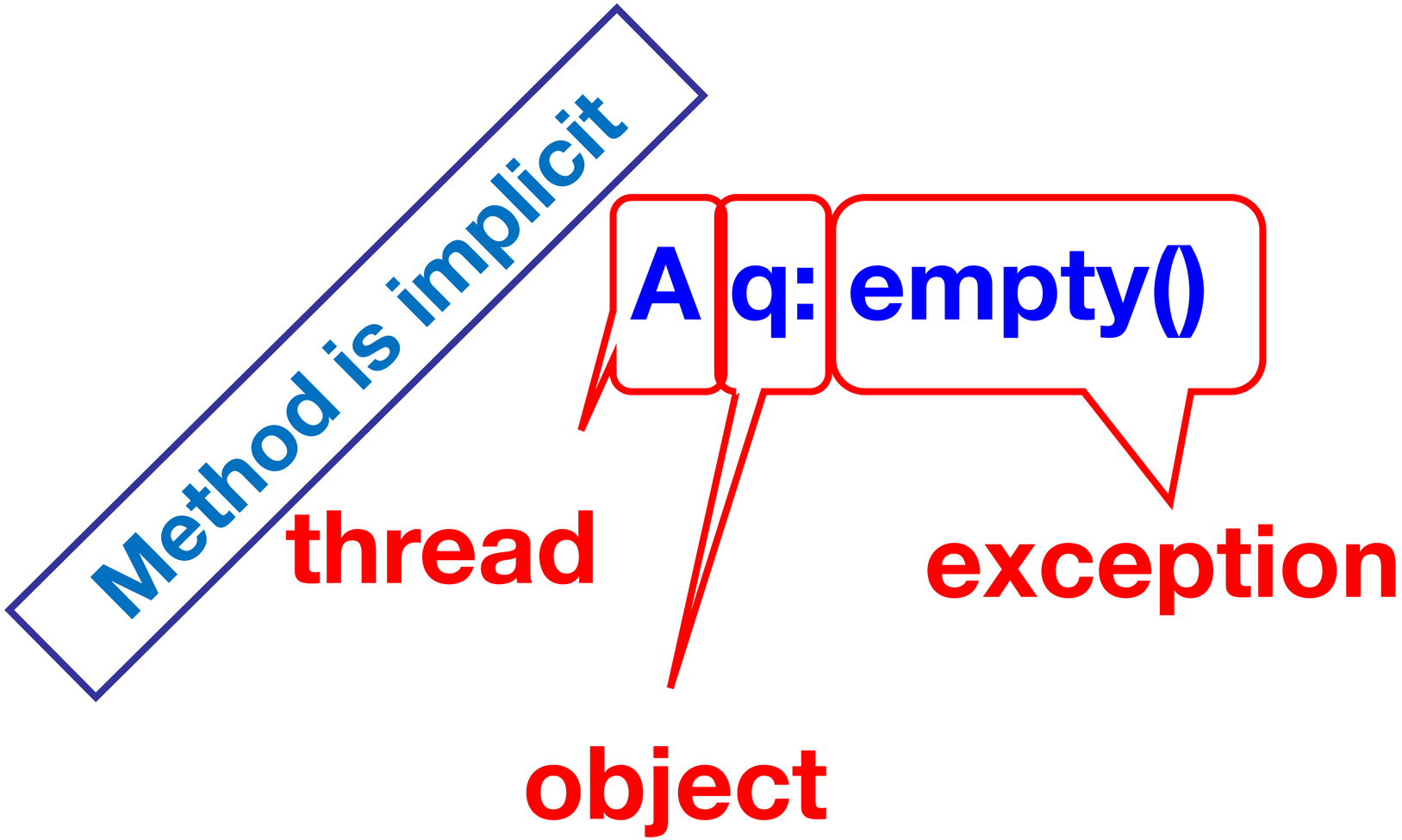
Response Notation



Response Notation



Response Notation



History — Describing an execution

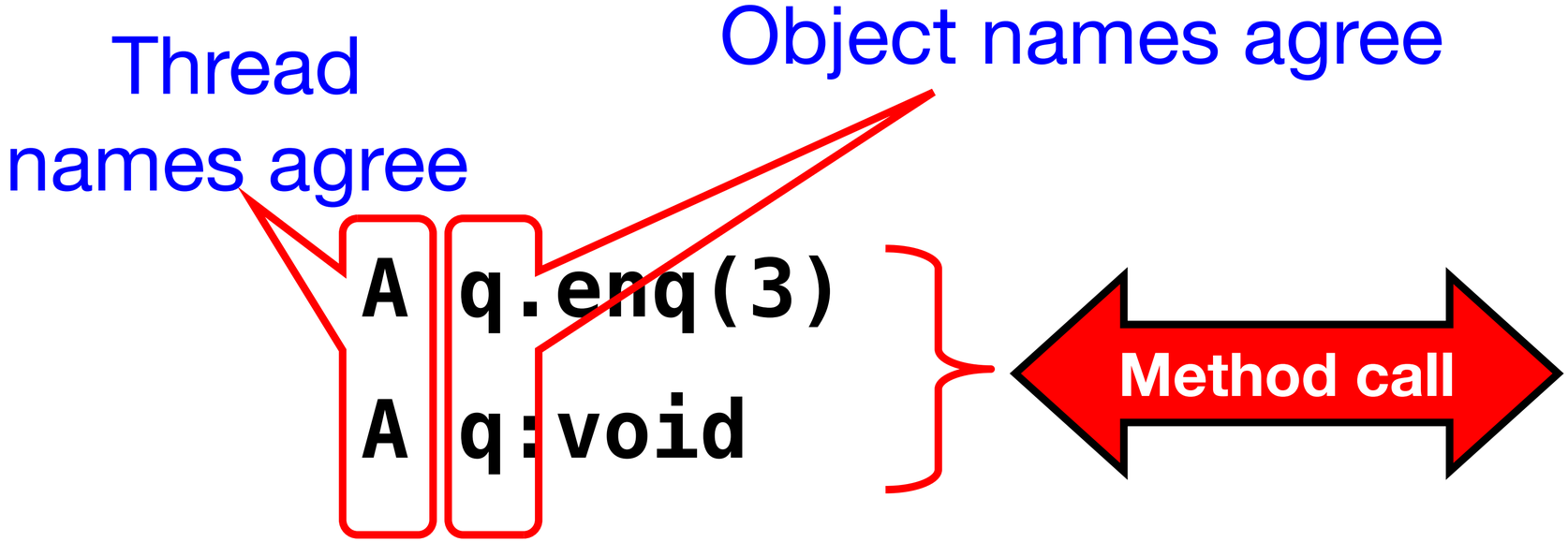
H =

```
A q.enq(3)
A q:void
A q.enq(5)
B p.enq(4)
B p:void
B q.deq()
B q:3
```

Sequence of invocations and responses

History — Describing an execution

- Invocation & response *match* if



Object Projections

H =

```
A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3
```

Object Projections

```
A q.enq(3)
A q:void
```

H|q =

```
B q.deq()
B q:3
```

Thread Projections

H =

```
A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3
```

Thread Projections

$H|B =$

```
B p.enq(4)
B p:void
B q.deq()
B q:3
```

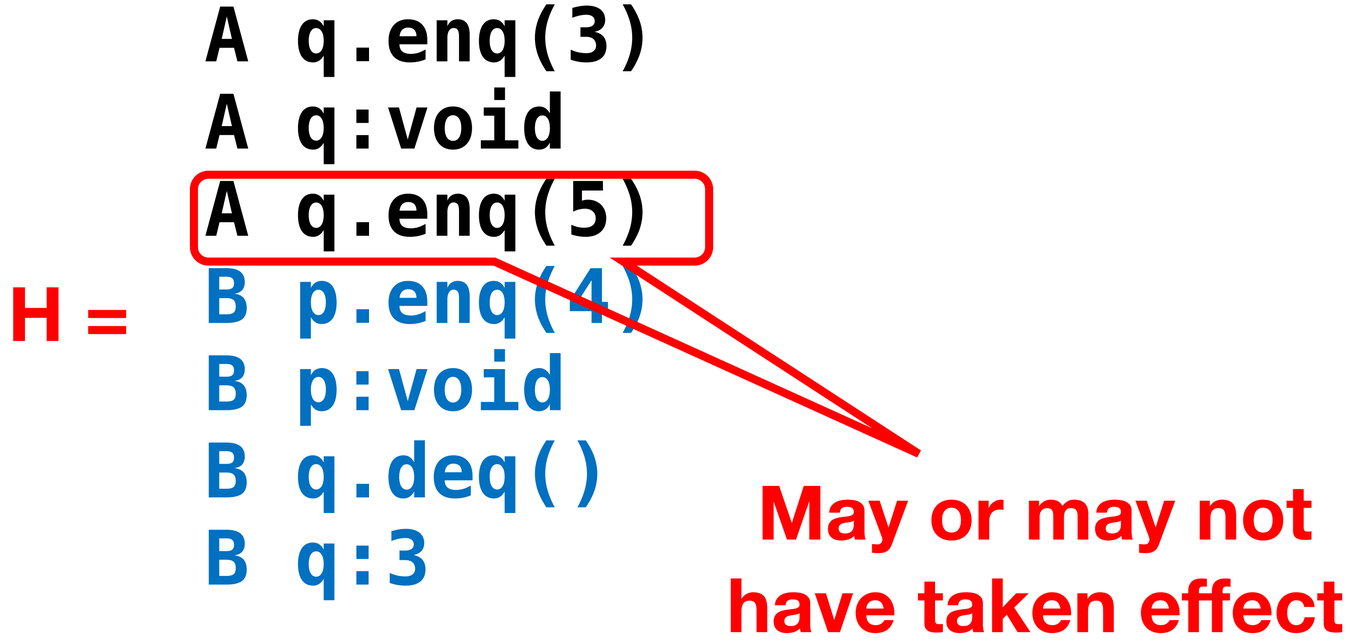
Complete Subhistory

H =

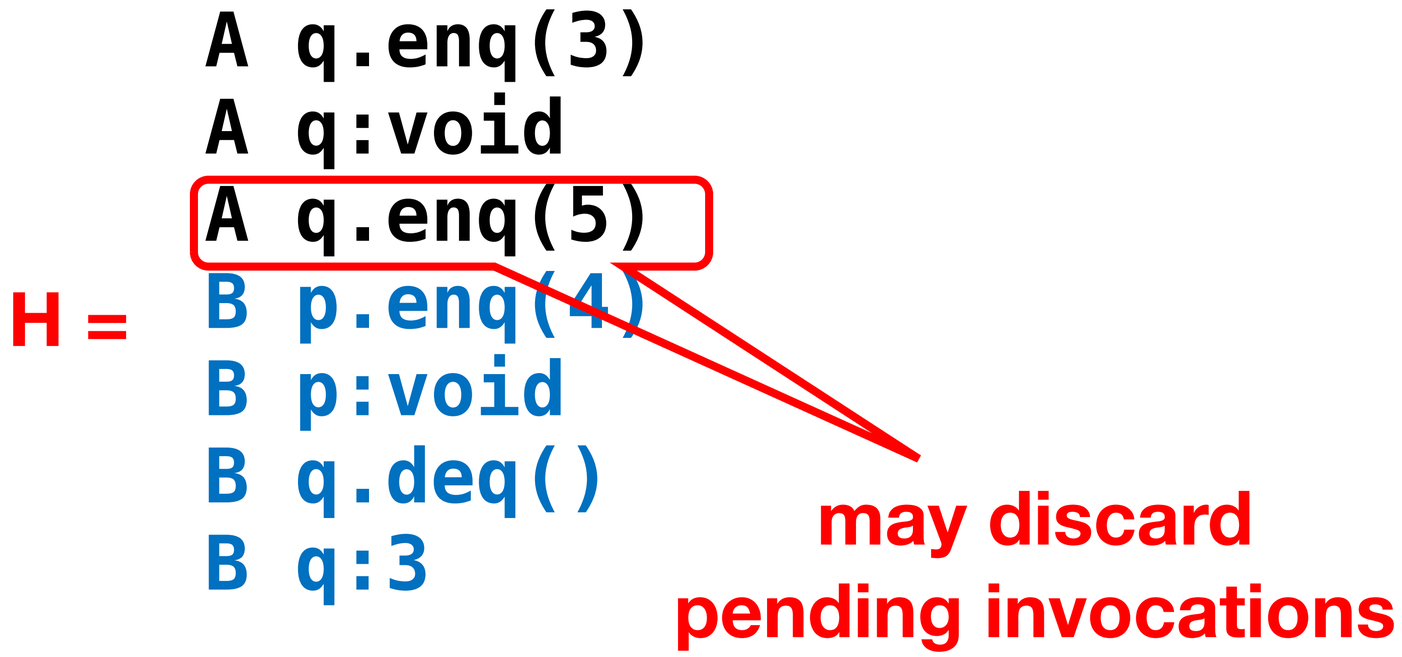
```
A q.enq(3)
A q:void
A q.enq(5)
B p.enq(4)
B p:void
B q.deq()
B q:3
```

An invocation is *pending* if it has no matching respnse

Complete Subhistory



Complete Subhistory



Complete Subhistory

```
A q.enq(3)
A q:void
```

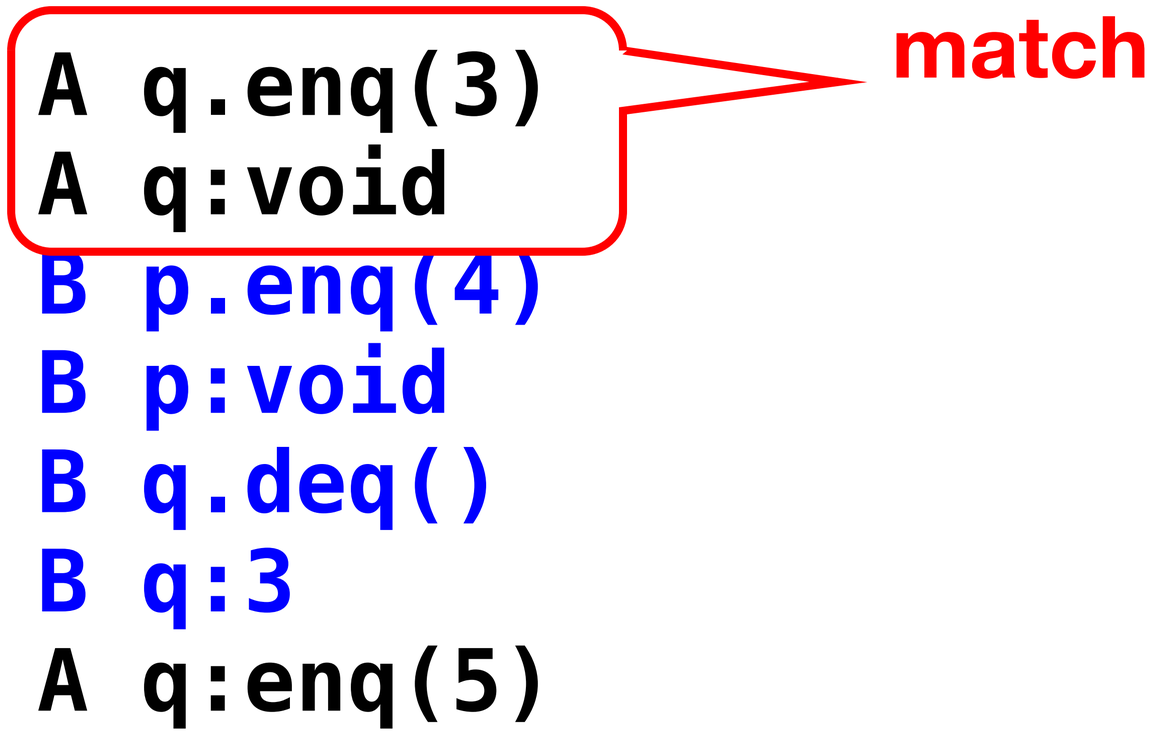
Complete(H) =

```
B p.enq(4)
B p:void
B q.deq()
B q:3
```

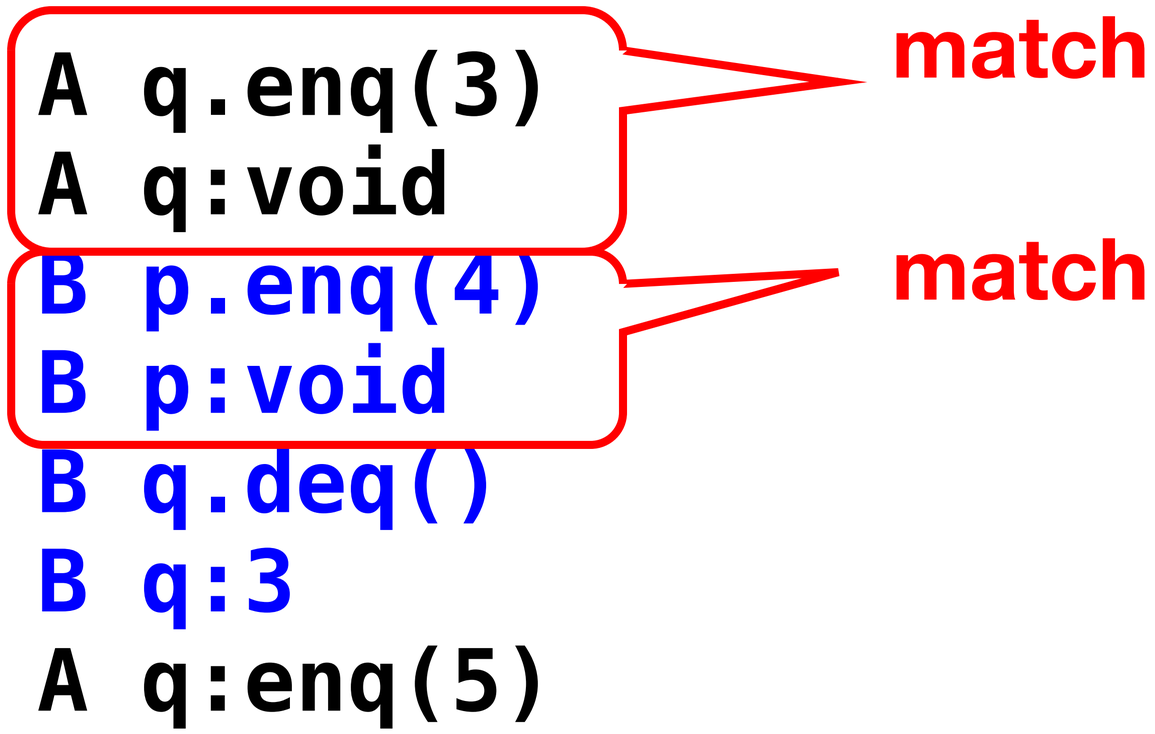
Sequential Histories

```
A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3
A q:enq(5)
```

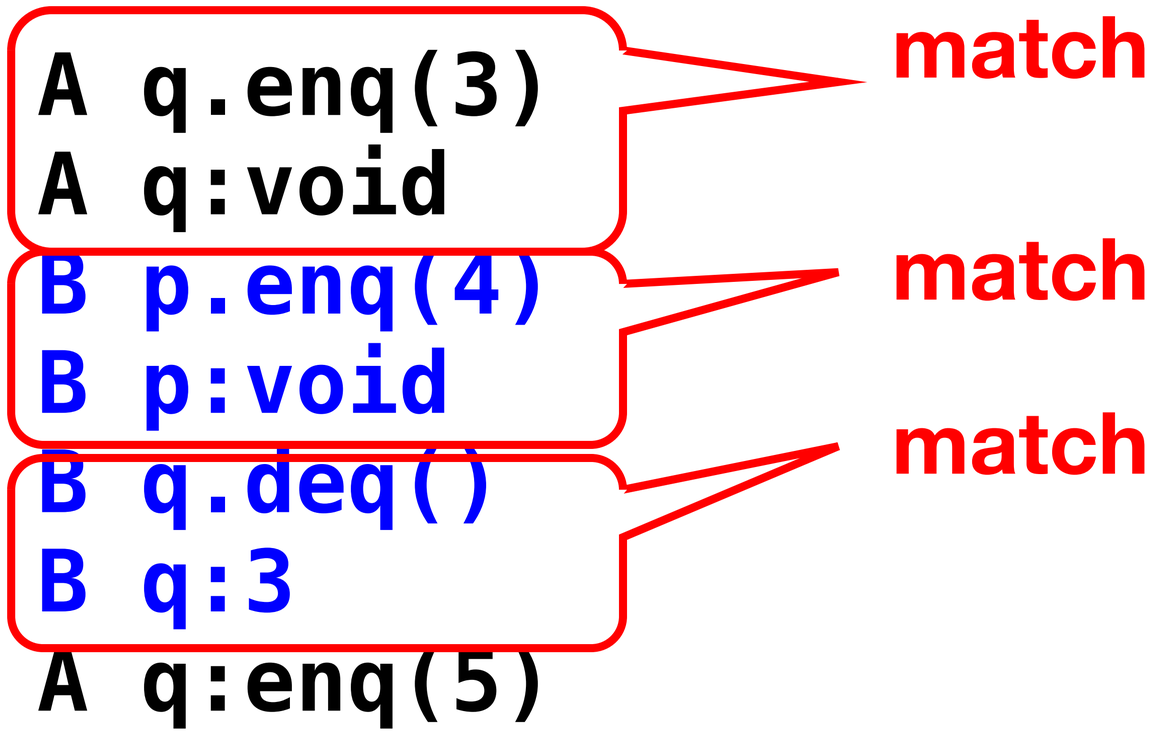
Sequential Histories



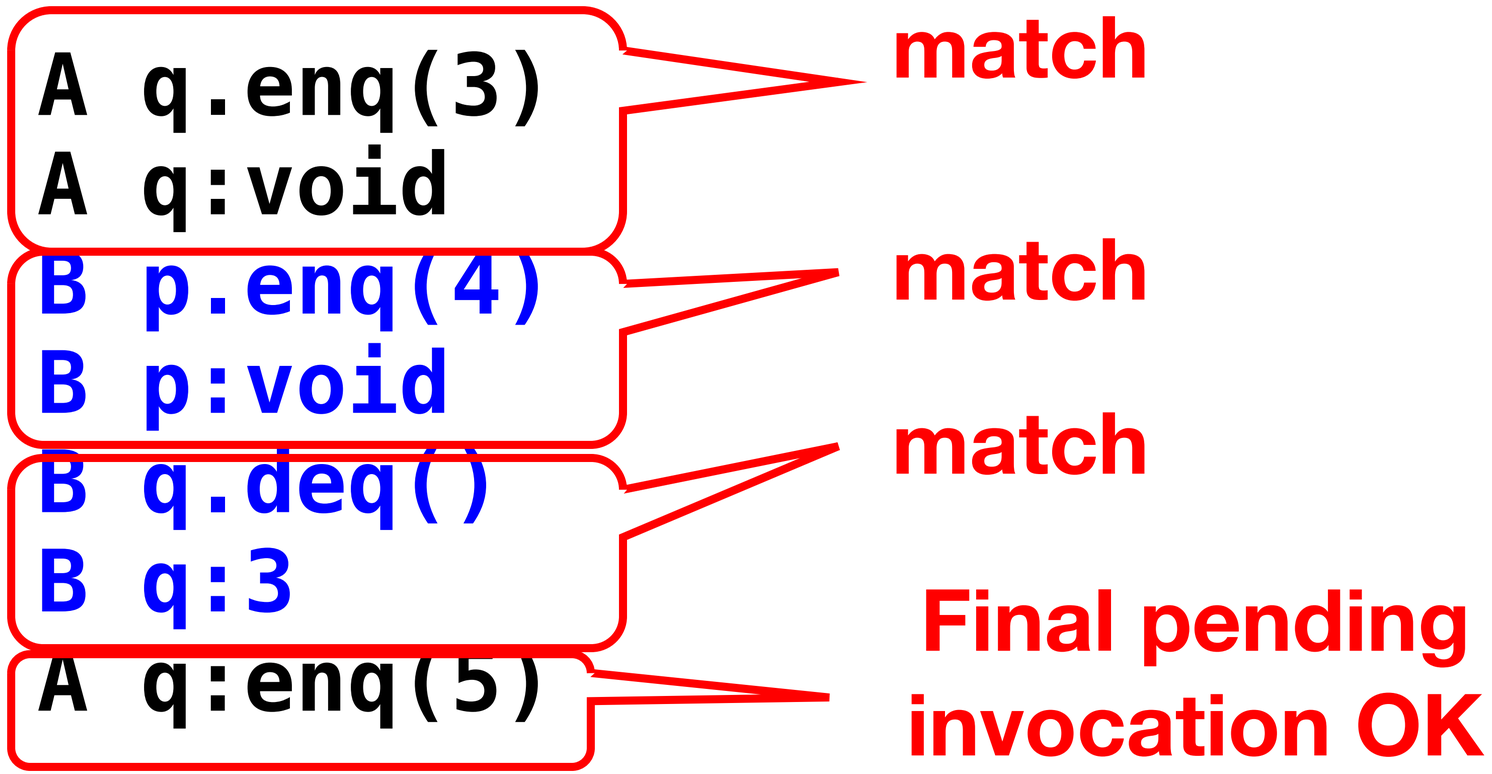
Sequential Histories



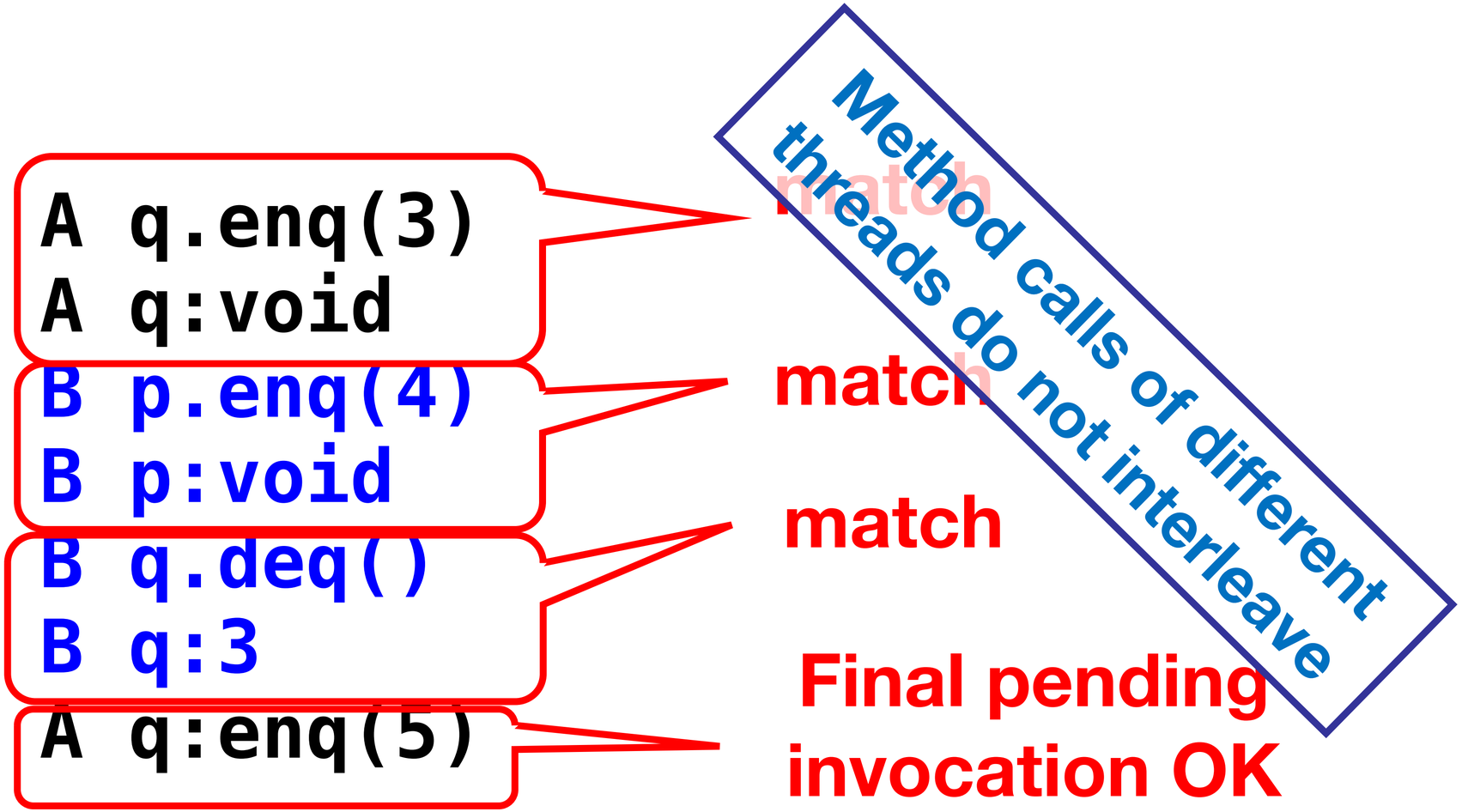
Sequential Histories



Sequential Histories



Sequential Histories



Well-formed Histories

```
     A q.enq(3)
     B p.enq(4)
     B p:void
H=   B q.deq()
     A q:void
     B q:3
```

Well-formed Histories

Per-thread projections sequential

H=

```
A q.enq(3)
B p.enq(4)
B p:void
B q.deq()
A q:void
B q:3
```

H|B=

```
B p.enq(4)
B p:void
B q.deq()
B q:3
```

Well-formed Histories

Per-thread projections sequential

H=

```
A q.enq(3)
B p.enq(4)
B p:void
B q.deq()
A q:void
B q:3
```

H|B=

```
B p.enq(4)
B p:void
B q.deq()
B q:3
```

H|A=

```
A q.enq(3)
A q:void
```

Equivalent Histories

Threads see the same thing in both

$$\begin{cases} H|A = G|A \\ H|B = G|B \end{cases}$$

H=

```
A q.enq(3)
B p.enq(4)
B p:void
B q.deq()
A q:void
B q:3
```

G=

```
A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3
```

Sequential Specifications

- A sequential ***specification*** is some way of telling whether a
  - Single-thread, single-object history is ***legal***
- For example:
  - Pre and post-conditions
  - But plenty of other techniques exist …

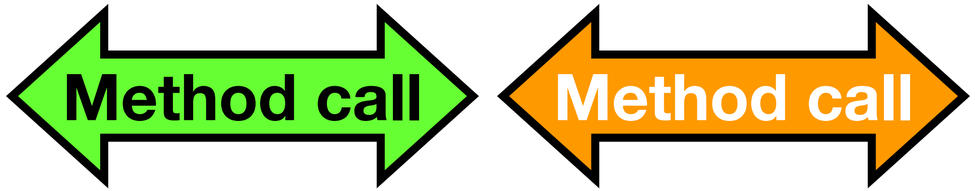
Legal Histories

- A sequential *(multi-object)* history H is *legal* if
  - For every object **x**
  - **H|x** is in the ***sequential spec*** for **x**

Precedence

```
A q.enq(3)
B p.enq(4)
B p.void
A q:void
B q.deq()
B q:3
```

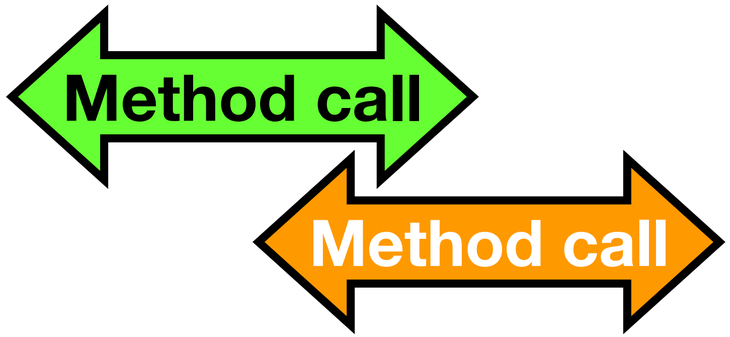
A method call precedes another if response event precedes invocation event



Non-Precedence

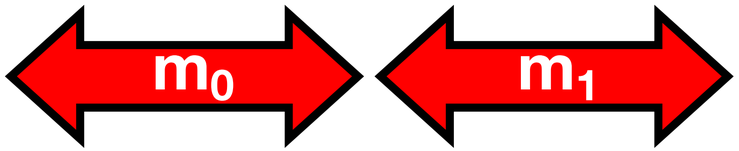
```
A q.enq(3)
B p.enq(4)
B p.void
B q.deq()
A q:void
B q:3
```

Some method calls
overlap one another



Notation

- Given
  - History **H**
  - method executions $\mathbf{m_0}$ and $\mathbf{m_1}$ in **H**
- We say $\mathbf{m_0} \rightarrow_{\mathbf{H}} \mathbf{m_1}$, if
  - $\mathbf{m_0}$ precedes $\mathbf{m_1}$



- Relation $\mathbf{m_0} \rightarrow_{\mathbf{H}} \mathbf{m_1}$ is a
  - Partial order
  - Total order if **H** is sequential

Linearizability

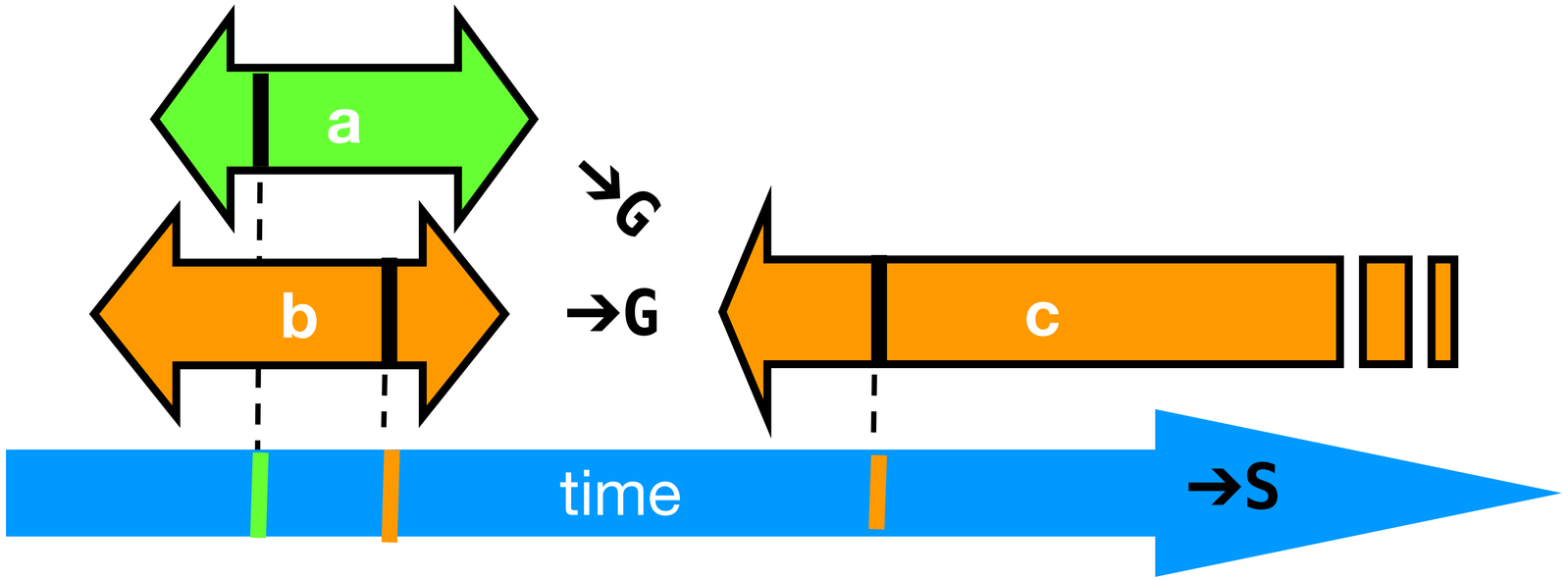
- History H is ***linearizable*** if it can be extended to **G** by
  - Appending zero or more responses to pending invocations
  - Discarding other pending invocations
- So that G is equivalent to
  - Legal sequential history **S**
  - where $\rightarrow_G \subset \rightarrow_S$

Remarks on Linearizability

- Some pending invocations
  - Took effect, so keep them
  - Discard the rest
- Condition $\rightarrow_G \subset \rightarrow_S$
  - Means that **S** respects “real-time order” of **G**

Ensuring →G ⊂ →S

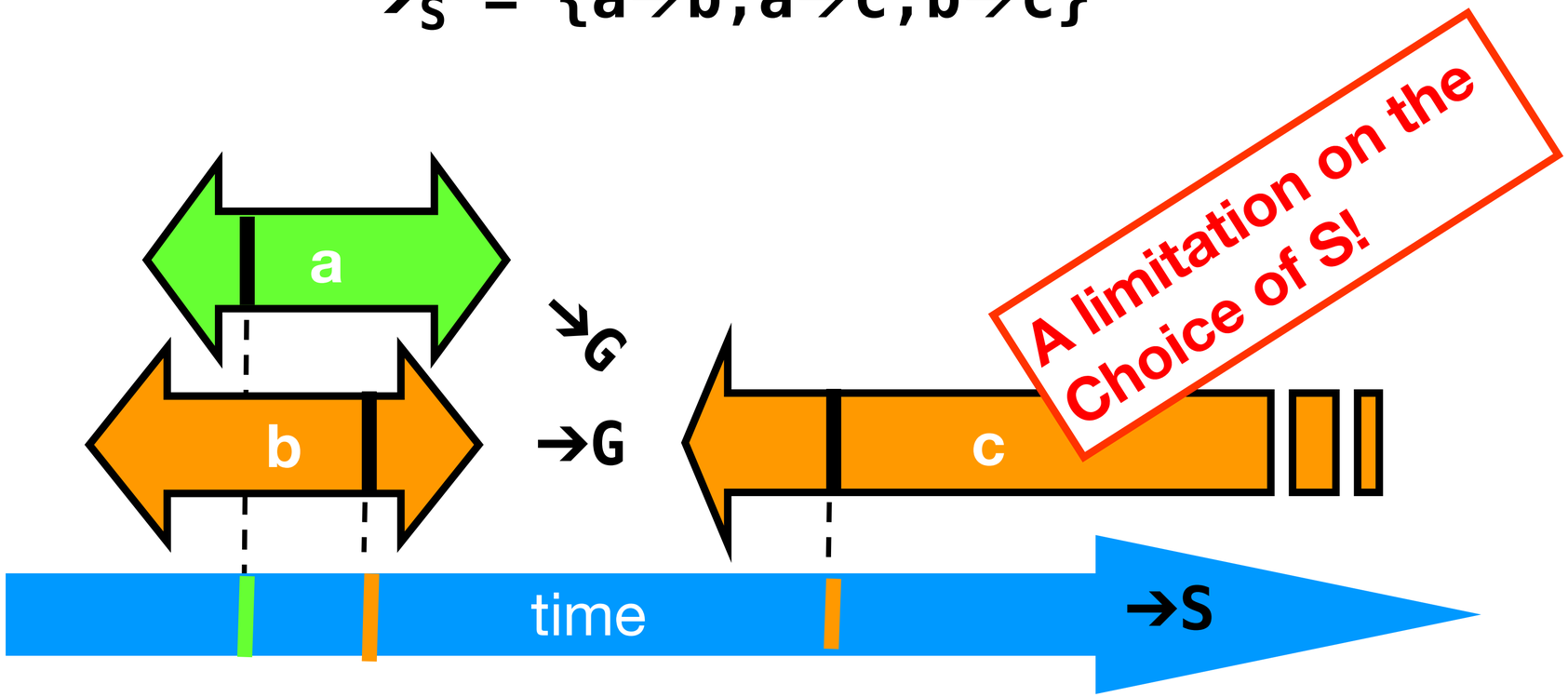
$$\rightarrow_G = \{a \rightarrow c, b \rightarrow c\}$$
$$\rightarrow_S = \{a \rightarrow b, a \rightarrow c, b \rightarrow c\}$$



Ensuring →G ⊂ →S

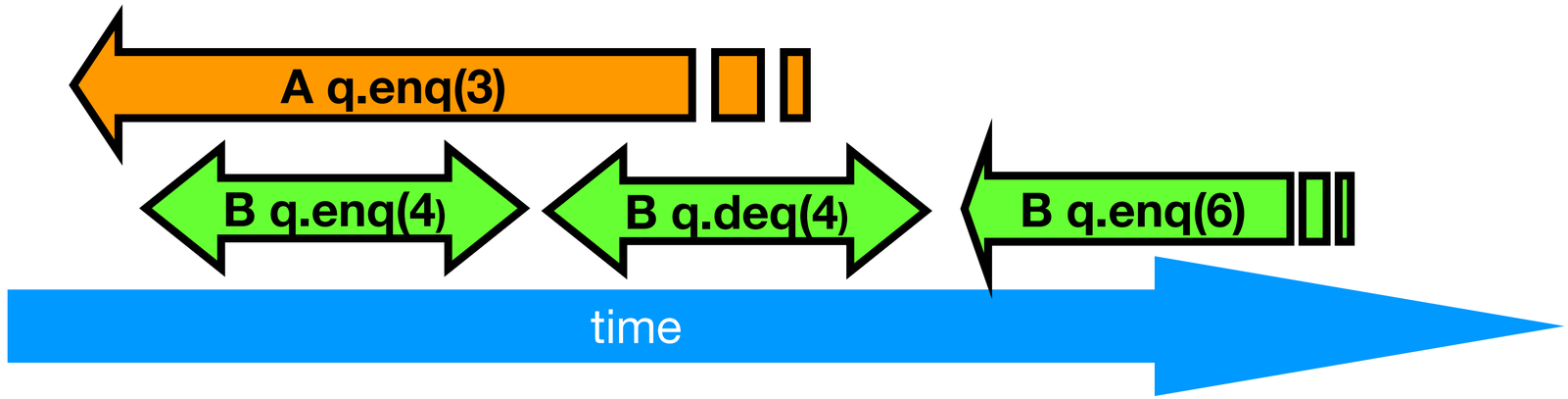
$\rightarrow_G = \{a \rightarrow c, b \rightarrow c\}$

$\rightarrow_S = \{a \rightarrow b, a \rightarrow c, b \rightarrow c\}$



Example

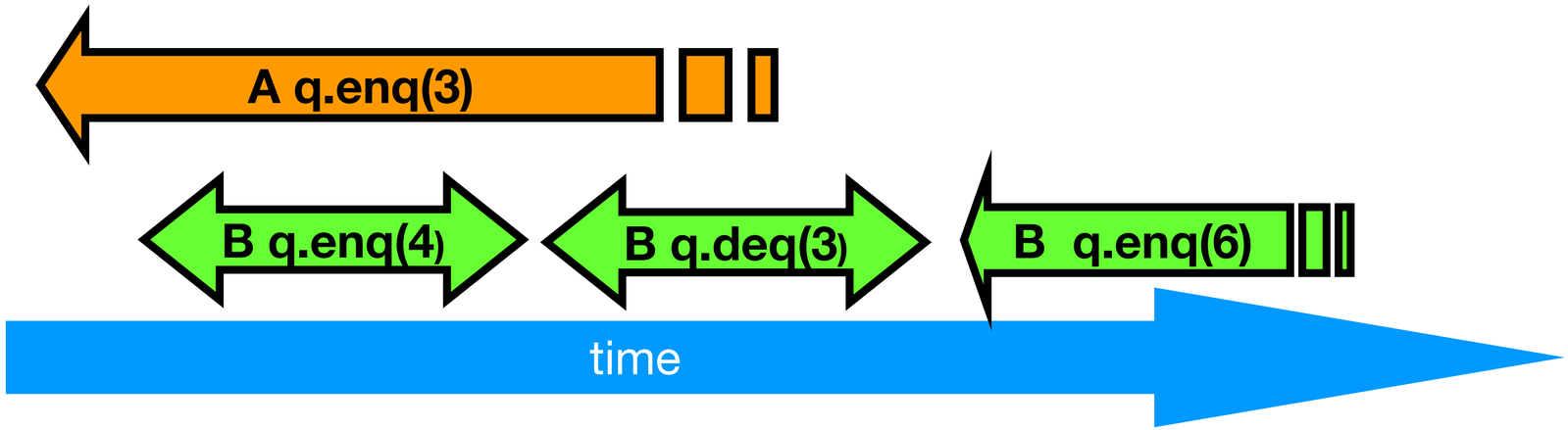
```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
B q:enq(6)
```



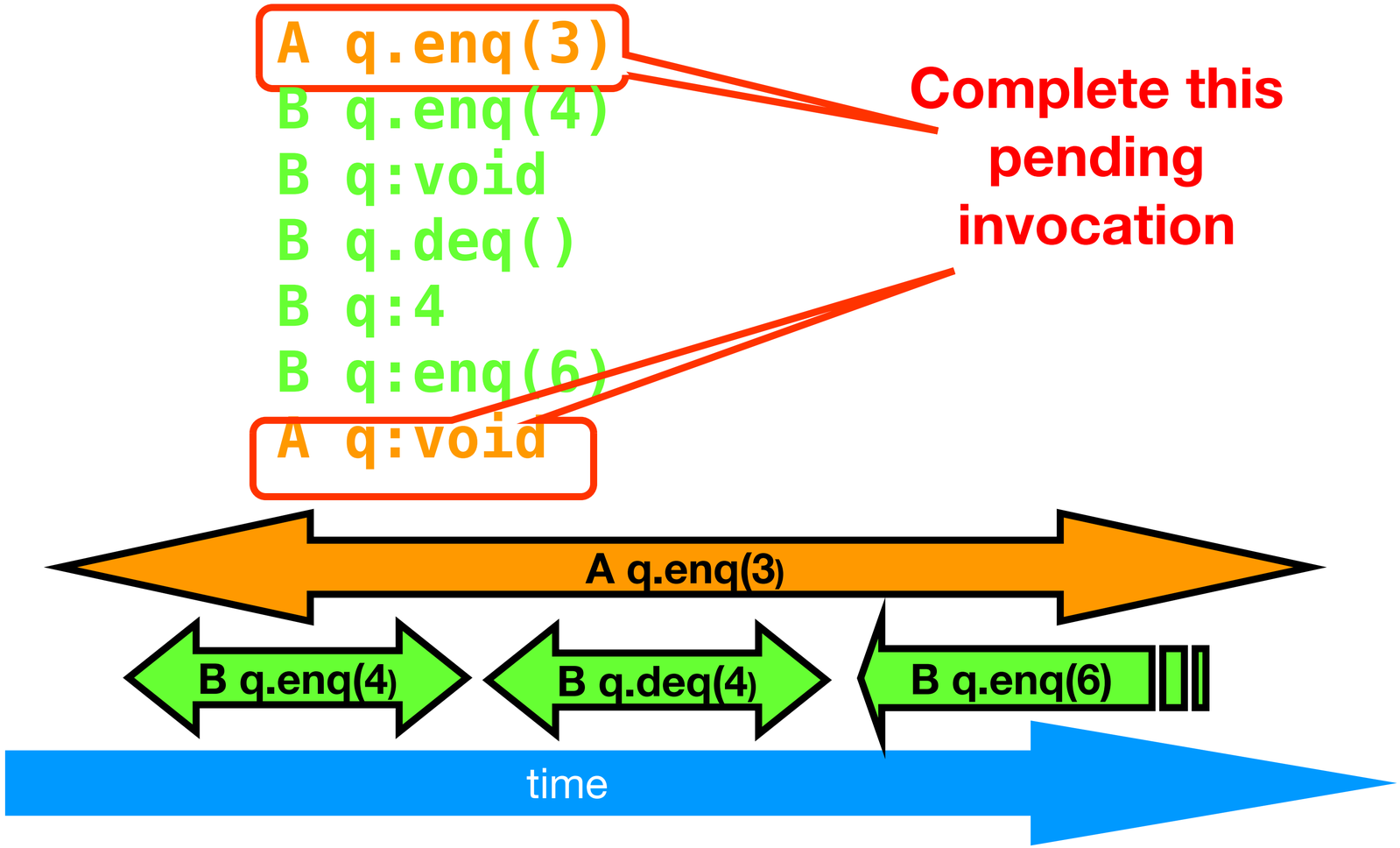
Example

```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
B q:enq(6)
```

Complete this pending invocation



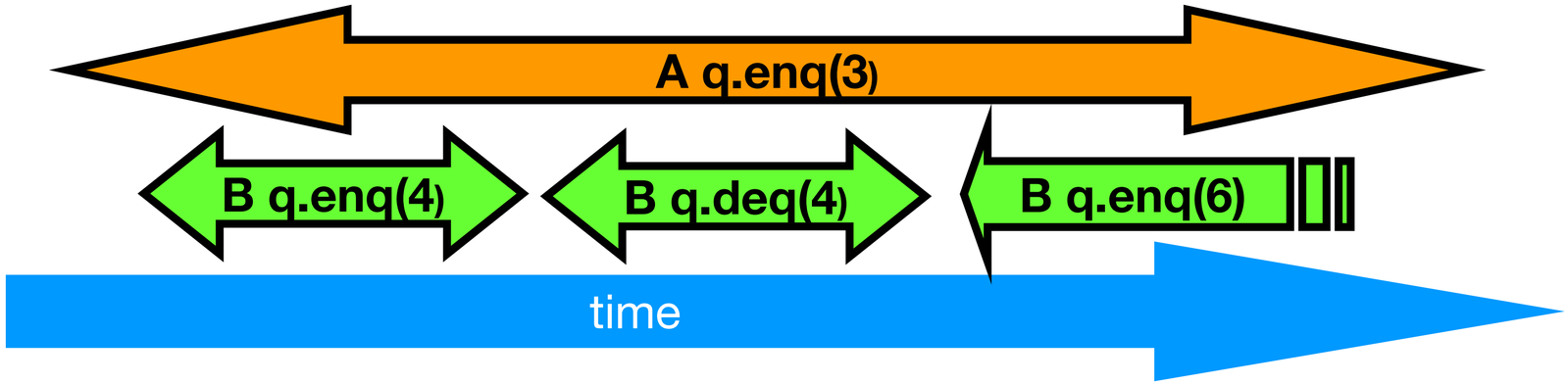
Example


A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
B q:enq(6)
A q:void
Complete this pending invocation
A q.enq(3)
B q.enq(4)
B q.deq(4)
B q.enq(6)
time

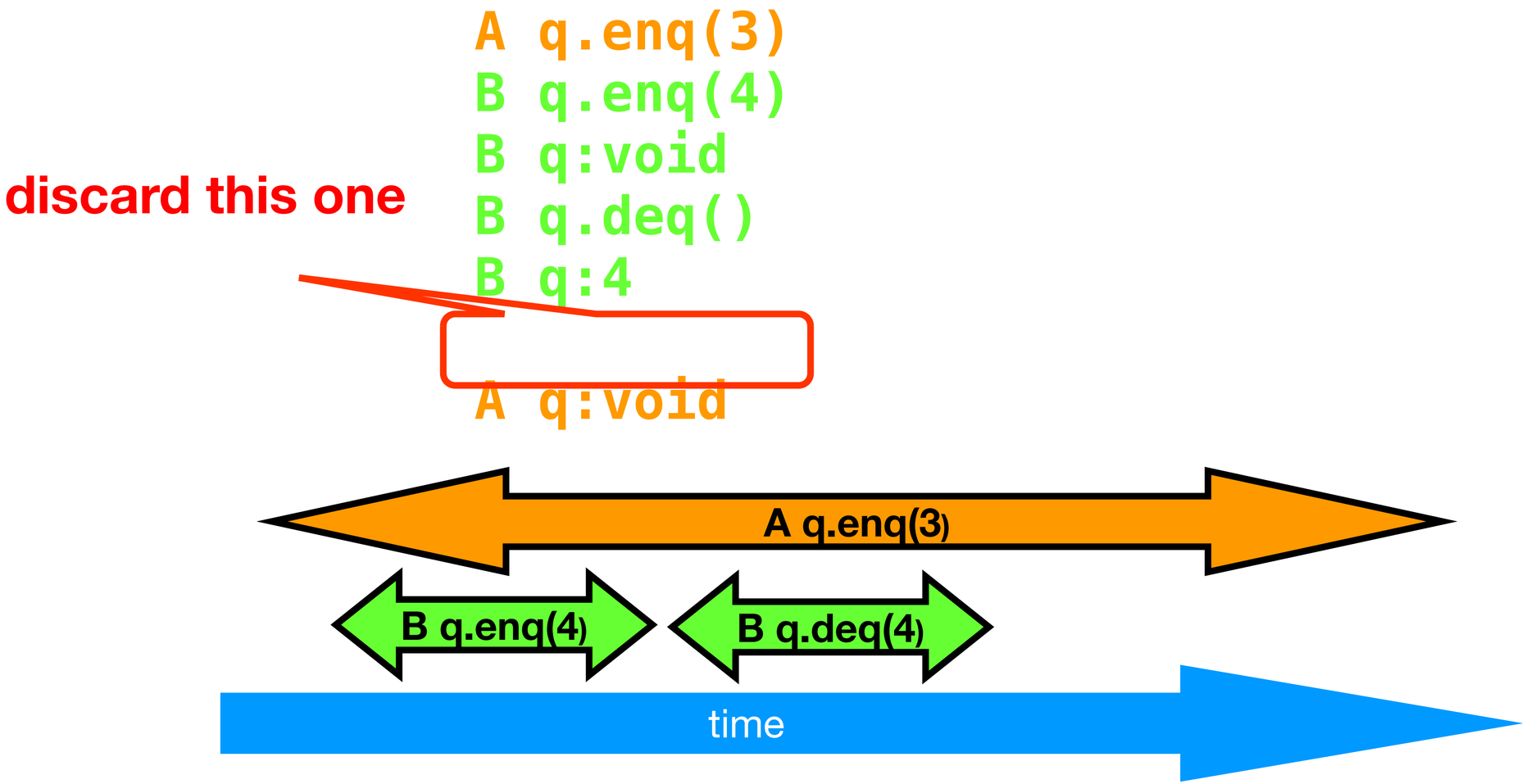
Example

discard this one

```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
B q:enq(6)
A q:void
```

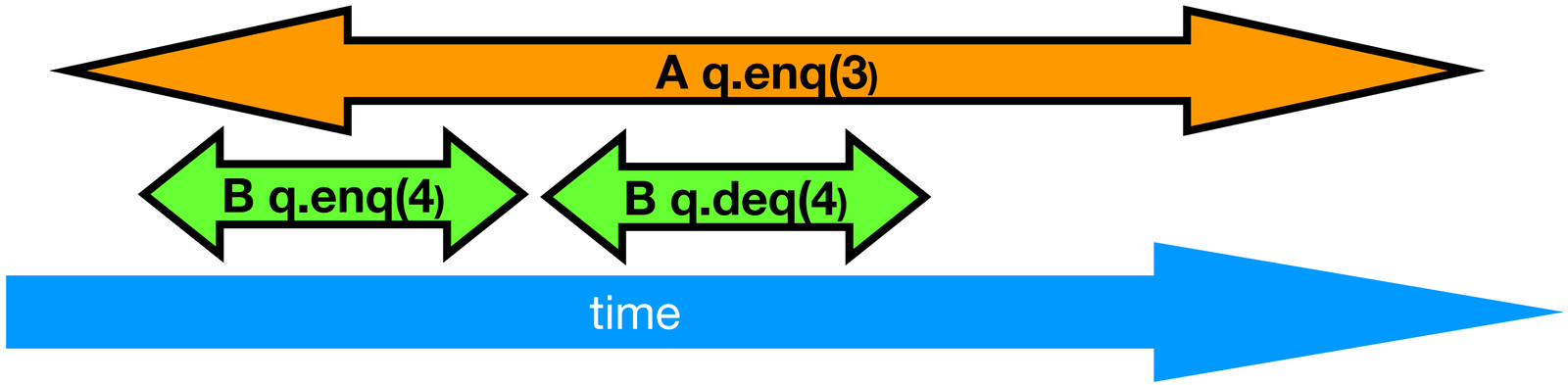


Example


A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
discard this one
A q:void
A q.enq(3)
B q.enq(4)
B q.deq(4)
time

Example

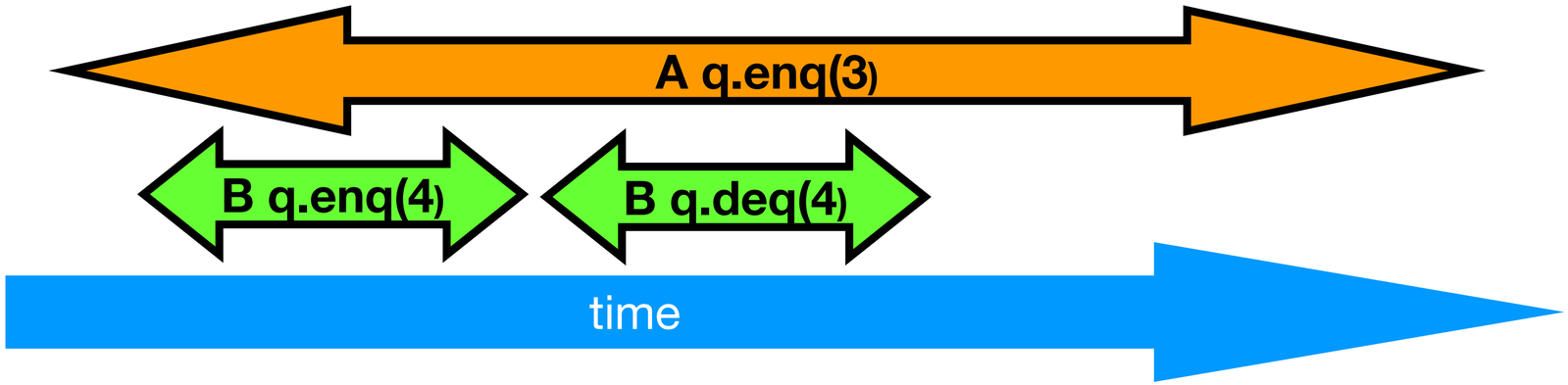
```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
A q:void
```



Example

```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
A q:void
```

```
B q.enq(4)
B q:void
A q.enq(3)
A q:void
B q.deq()
B q:4
```

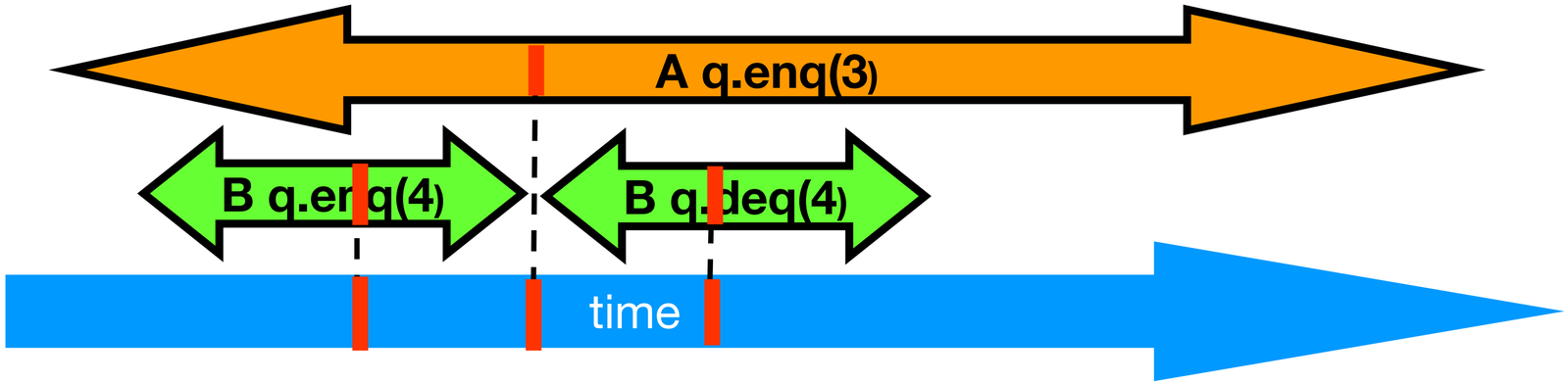


Example

```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
A q:void
```

Equivalent sequential history

```
B q.enq(4)
B q:void
A q.enq(3)
A q:void
B q.deq()
B q:4
```



Composability Theorem

- History H is linearizable if and only if
  - For every object x
  - H|x is linearizable
- We care about objects only!
  - (Materialism?)

Why does composability matter?

- Modularity
- Can prove linearizability of objects in isolation
- Can compose independently-implemented objects

Reasoning about Linearizability: Locking

```
let deq q =
  Mutex.lock q.lock;
  try
    if q.tail = q.head then
      raise Empty;
    match q.items.(q.head mod q.capacity) with
    | None -> assert false
    | Some x ->
        q.head <- q.head + 1;
        Mutex.unlock q.lock;
        x
  with e ->
    Mutex.unlock q.lock;
    raise e
```

Linearization points are when locks are released

More Reasoning — Wait-free

```
(** Enqueue - should be called by only ONE thread *)
let enq q x =
  (* Check if queue is full *)
  if q.tail - q.head = q.capacity then
    raise Full;
  (* Write to the array *)
  q.items.(q.tail mod q.capacity) <- Some x;
  (* Advance tail *)
  q.tail <- q.tail + 1
```

```
(** Dequeue - should be called by only ONE thread *)
let deq q =
  (* Check if queue is empty *)
  if q.tail = q.head then
    raise Empty;
  (* Read from the array *)
  match q.items.(q.head mod q.capacity) with
  | None -> assert false  (* Should never happen *)
  | Some x ->
      (* Advance head *)
      q.head <- q.head + 1;
      x
```

More Reasoning — Wait-free

```
(** Enqueue - should be called by only ONE thread *)
let enq q x =
  (* Check if queue is full *)
  if q.tail - q.head = q.capacity then
    raise Full;
  (* Write to the array *)
  q.items.(q.tail mod q.capacity) <- Some x;
  (* Advance tail *)
  q.tail <- q.tail + 1
```

```
(** Dequeue - should be called by only ONE thread *)
let deq q =
  (* Check if queue is empty *)
  if q.tail = q.head then
    raise Empty;
  (* Read from the array *)
  match q.items.(q.head mod q.capacity) with
  | None -> assert false  (* Should never happen *)
  | Some x ->
      (* Advance head *)
      q.head <- q.head + 1;
      x
```

Linearization order is order head and tail fields modified

More Reasoning — Wait-free

```
(** Enqueue - should be called by only ONE thread
let enq q x =
  (* Check if queue is full *)
  if q.tail - q.head = q.capacity then
    raise Full;
  (* Write to the array *)
  q.items.(q.tail mod q.capacity)
  (* Advance tail *)
  q.tail <- q.tail + 1
```

```
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=
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  raise Empty;
(* Read from the array *)
match q.items.(q.head mod q.capacity) with
| None -> assert false  (* Should never happen *)
| Some x ->
    (* Advance head *)
    q.head <- q.head + 1;
    x
```
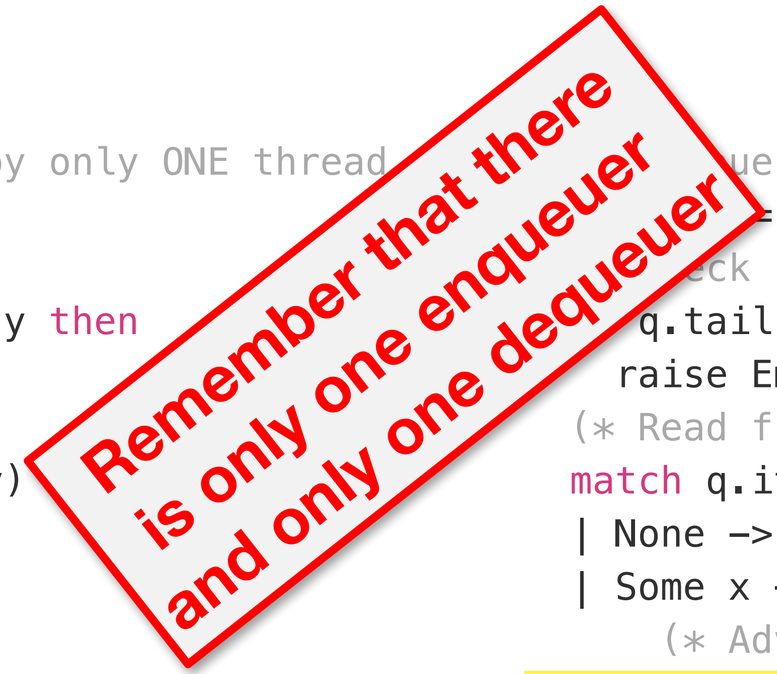



Linearization order is order head and tail fields modified

Finding linearisation points

- Identify one atomic step where the method “happens”
  - Critical section
  - Machine instruction
- Doesn’t always work
  - Might need to define several different steps for a given method
  - We will see this phenomenon in future lectures

Linearizability: Summary

- Powerful specification tool for shared objects
- Allows us to capture the notion of objects being “atomic”
- Don’t leave home without it

Alternative: Sequential Consistency

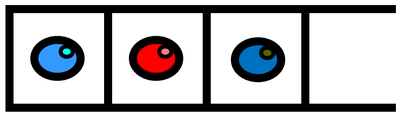
- History H is ***Sequentially Consistent*** if it can be extended to **G** by
  - Appending zero or more responses to pending invocations
  - Discarding other pending invocations
- So that G is equivalent to
  - Legal sequential history **S**
  - ~~where $\rightarrow_G \subset \rightarrow_S$~~

Differs from Linearizability

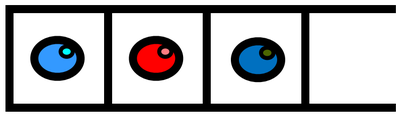
Sequential Consistency

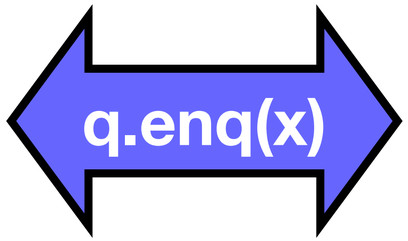
- No need to preserve real-time order
  - ***Cannot*** re-order operations done by the same thread
  - ***Can*** re-order non-overlapping operations done by different threads
- Often used to describe multiprocessor memory architectures

Example

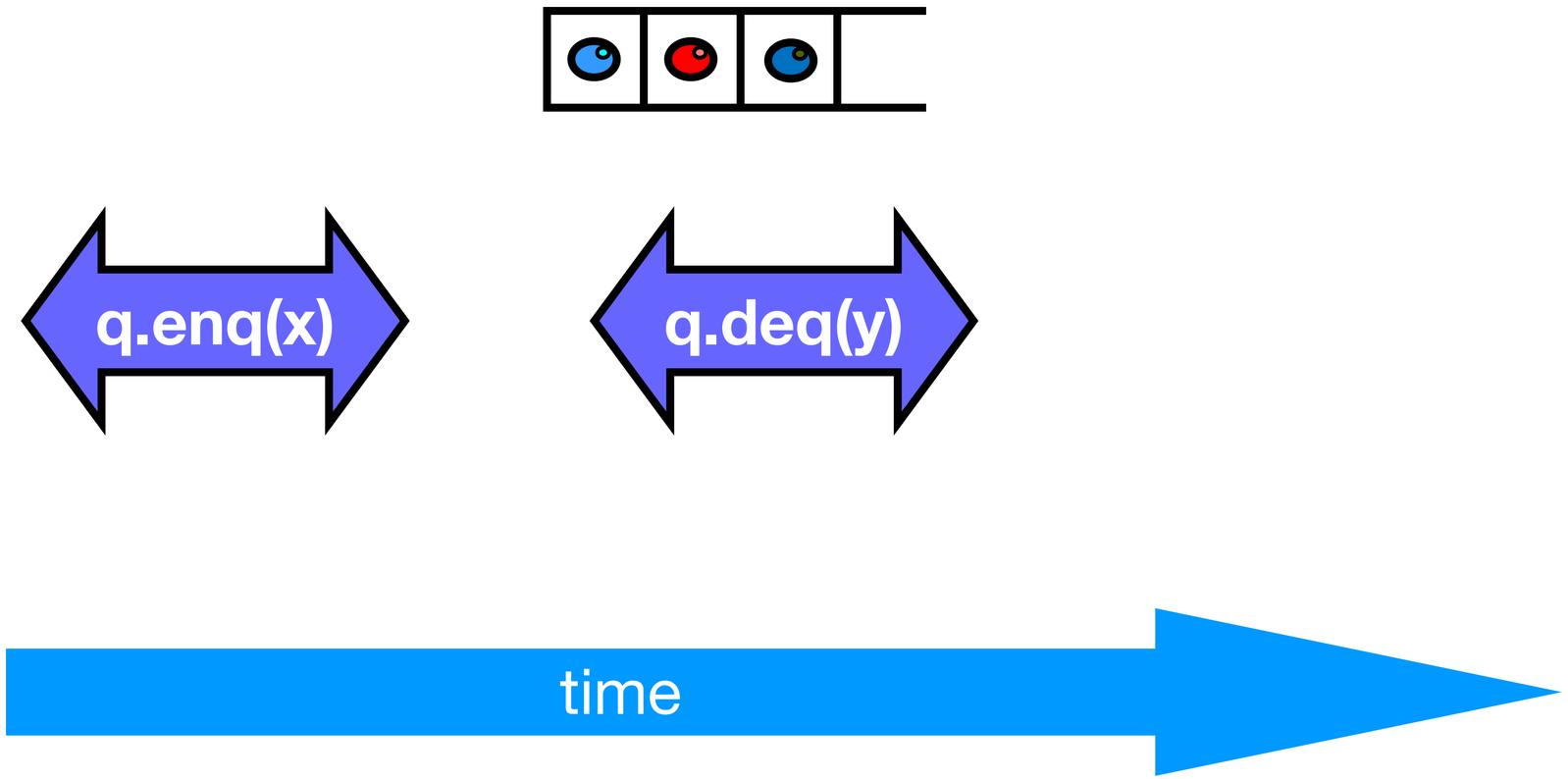


Example

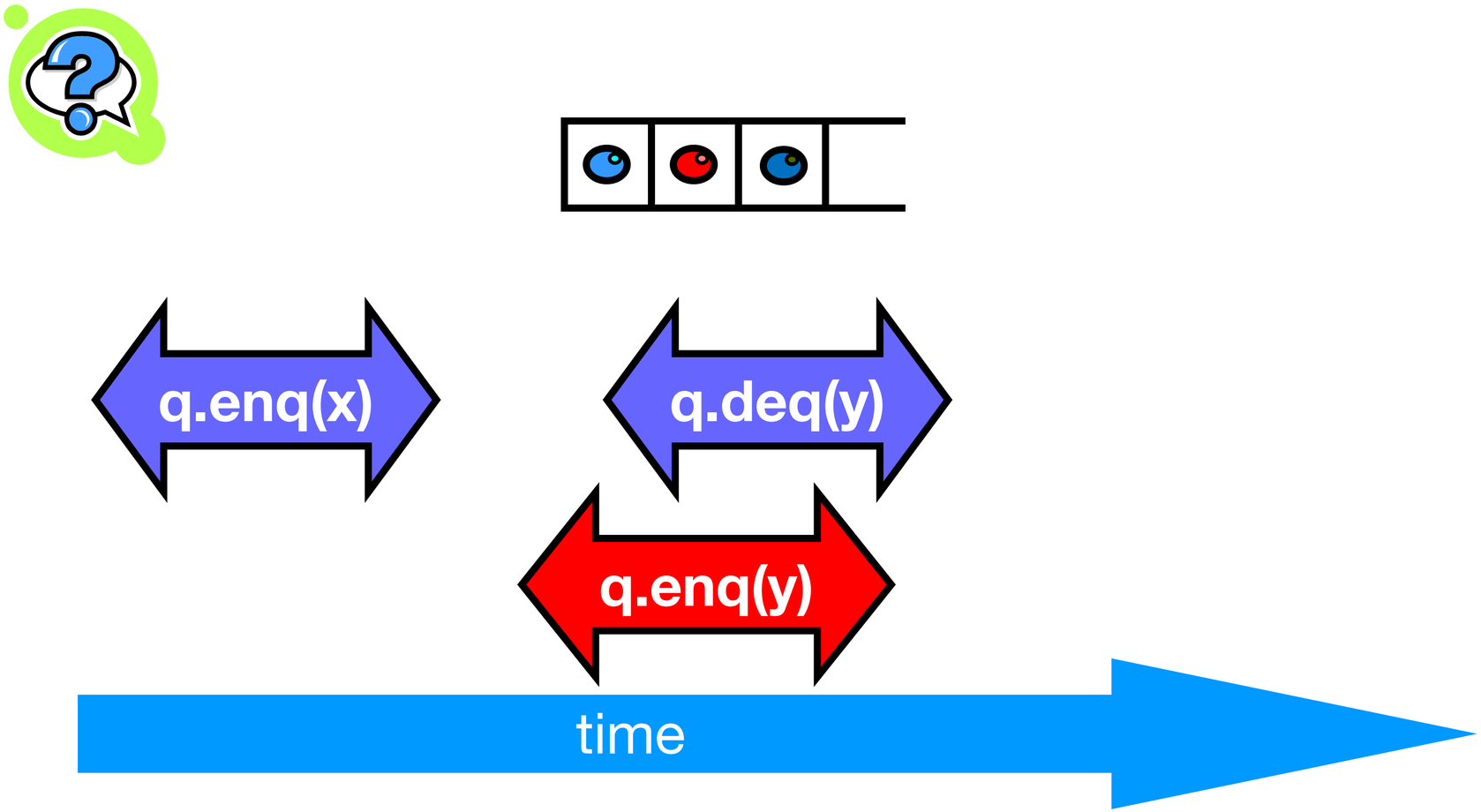




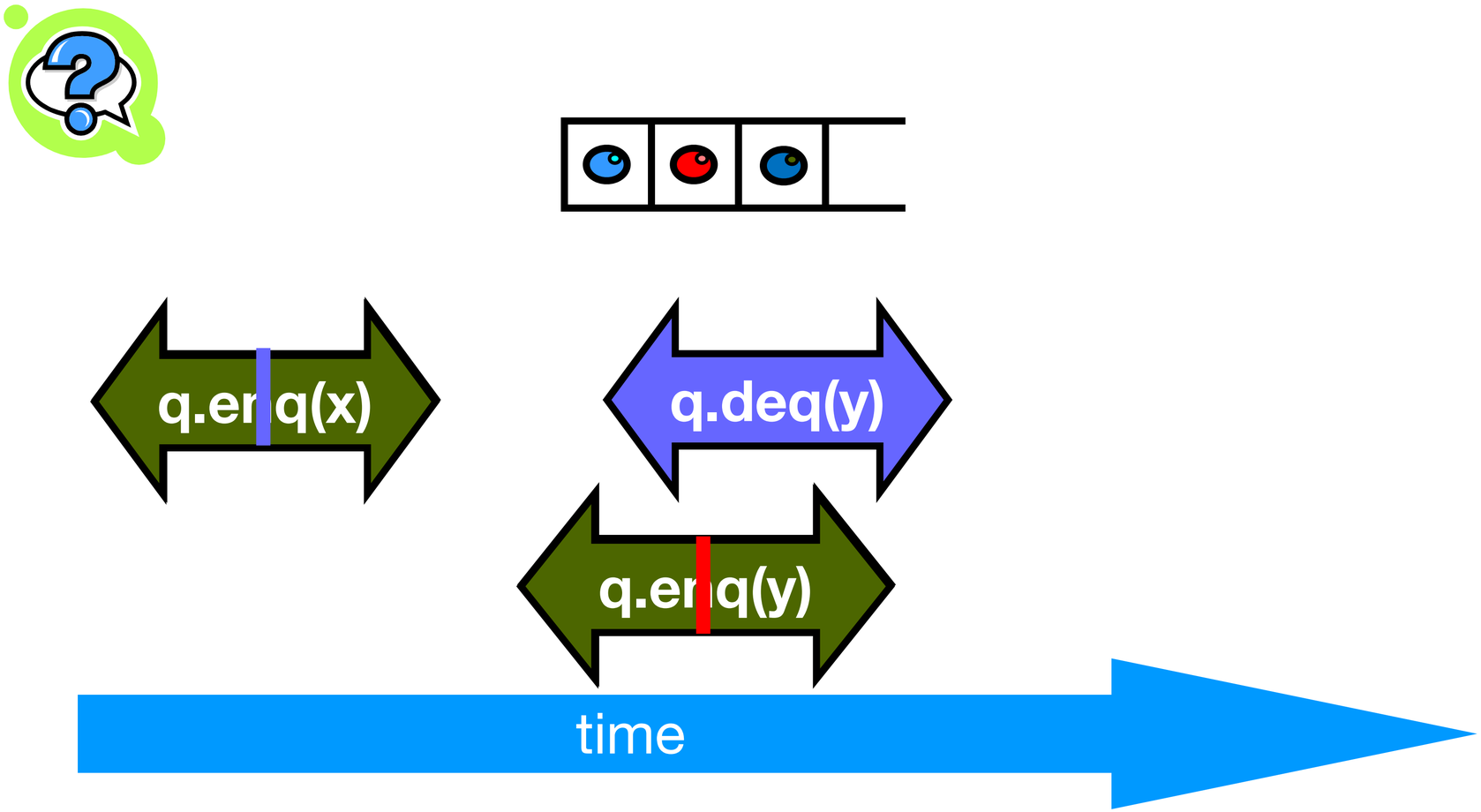
Example



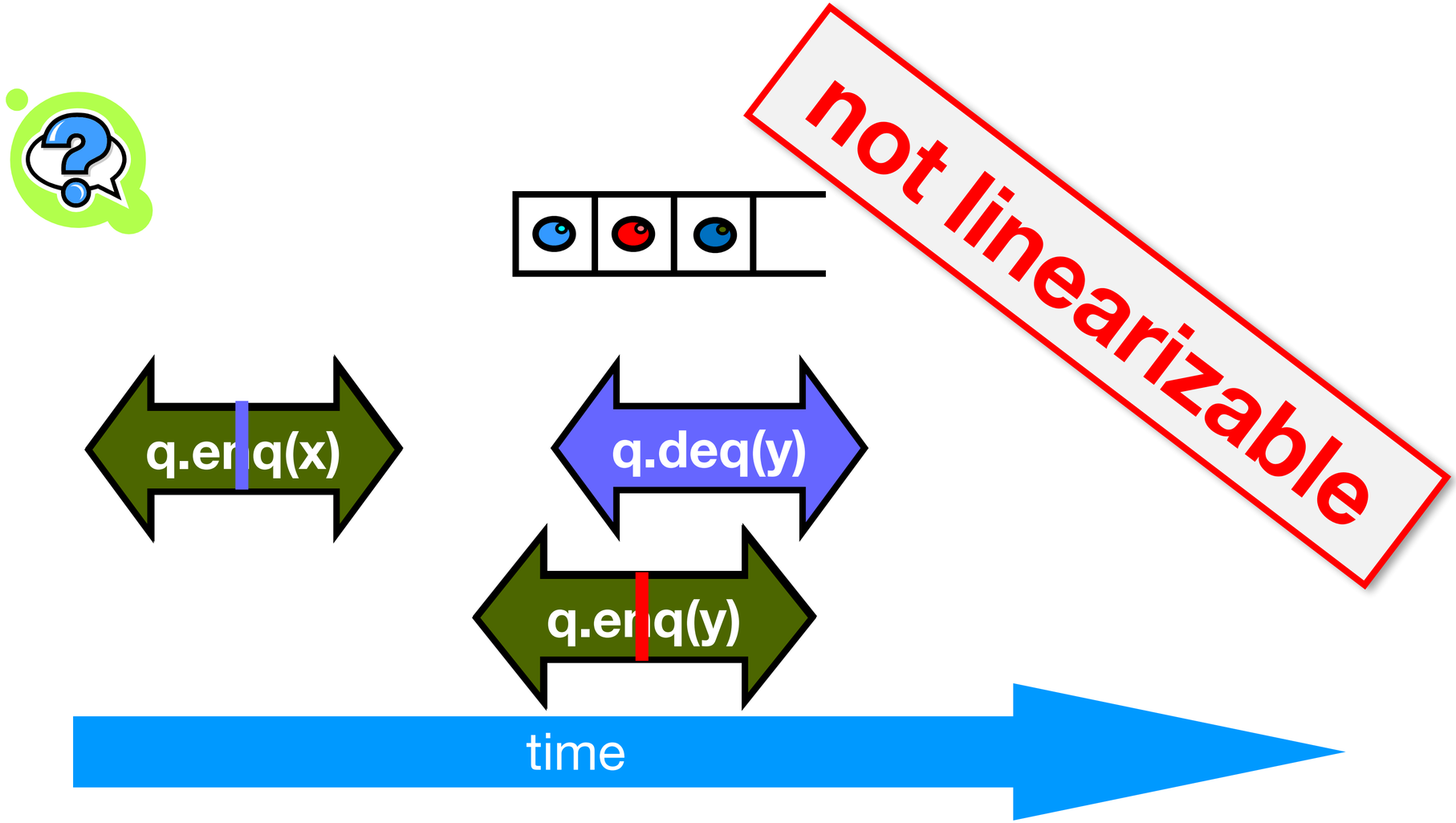
Example



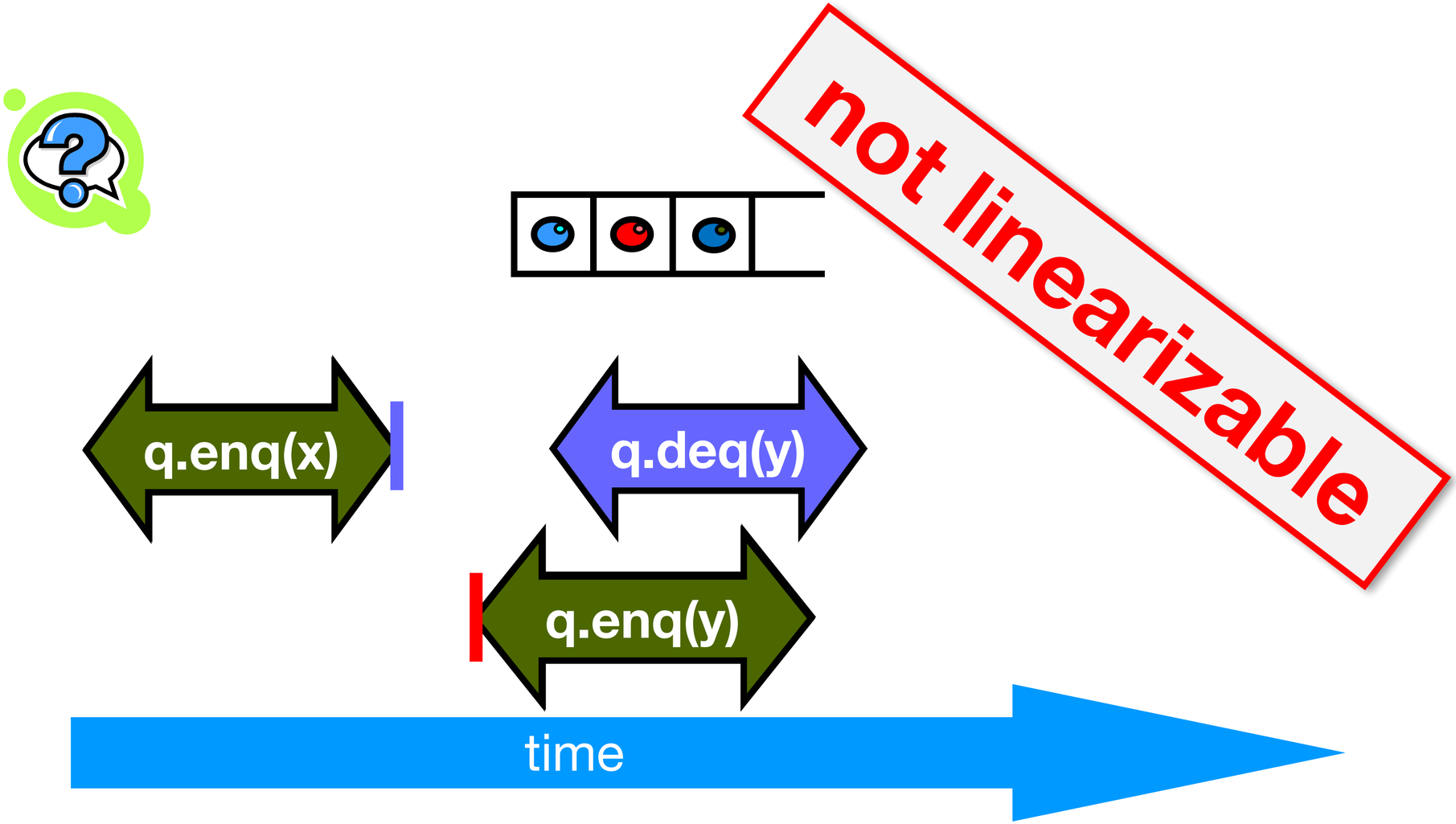
Example



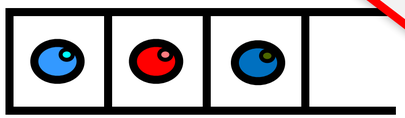
Example

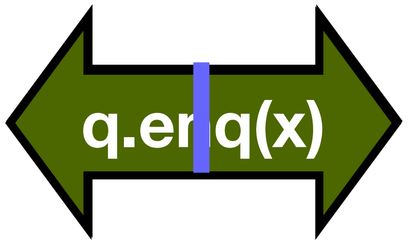


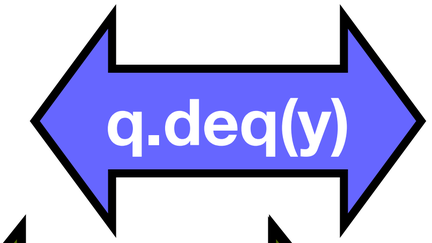
Example

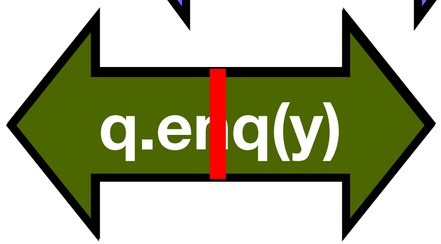


Example



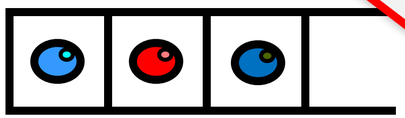






Yet Sequentially Consistent

Example



Yet Sequentially Consistent

q.enq(x)

q.deq(y)

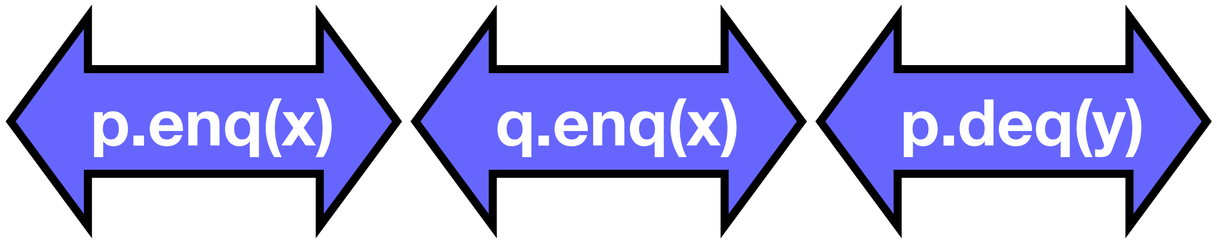
q.enq(y)

time

Theorem

Sequential Consistency is not Composable

FIFO Queue Example



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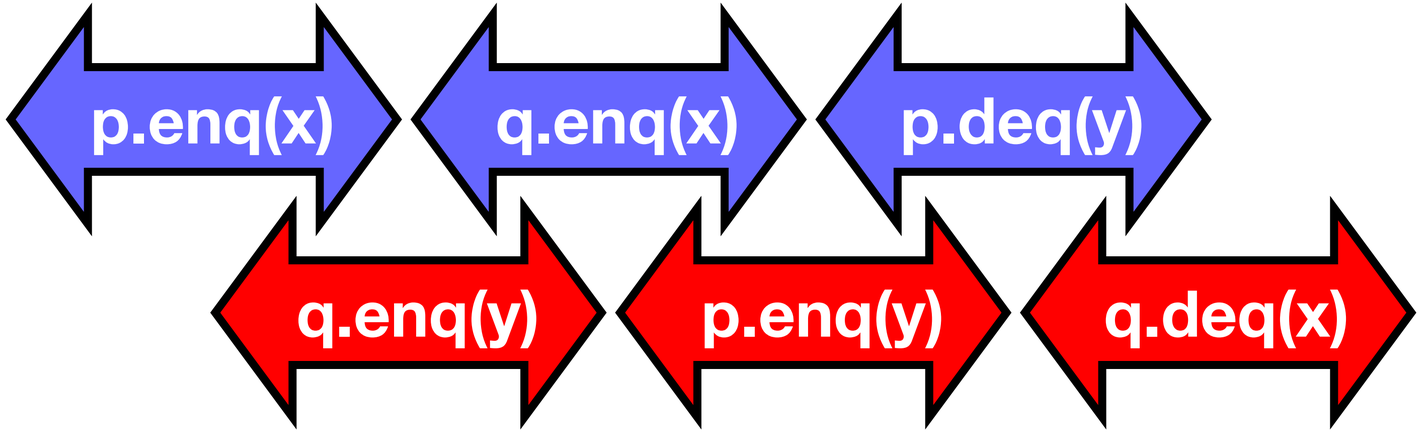

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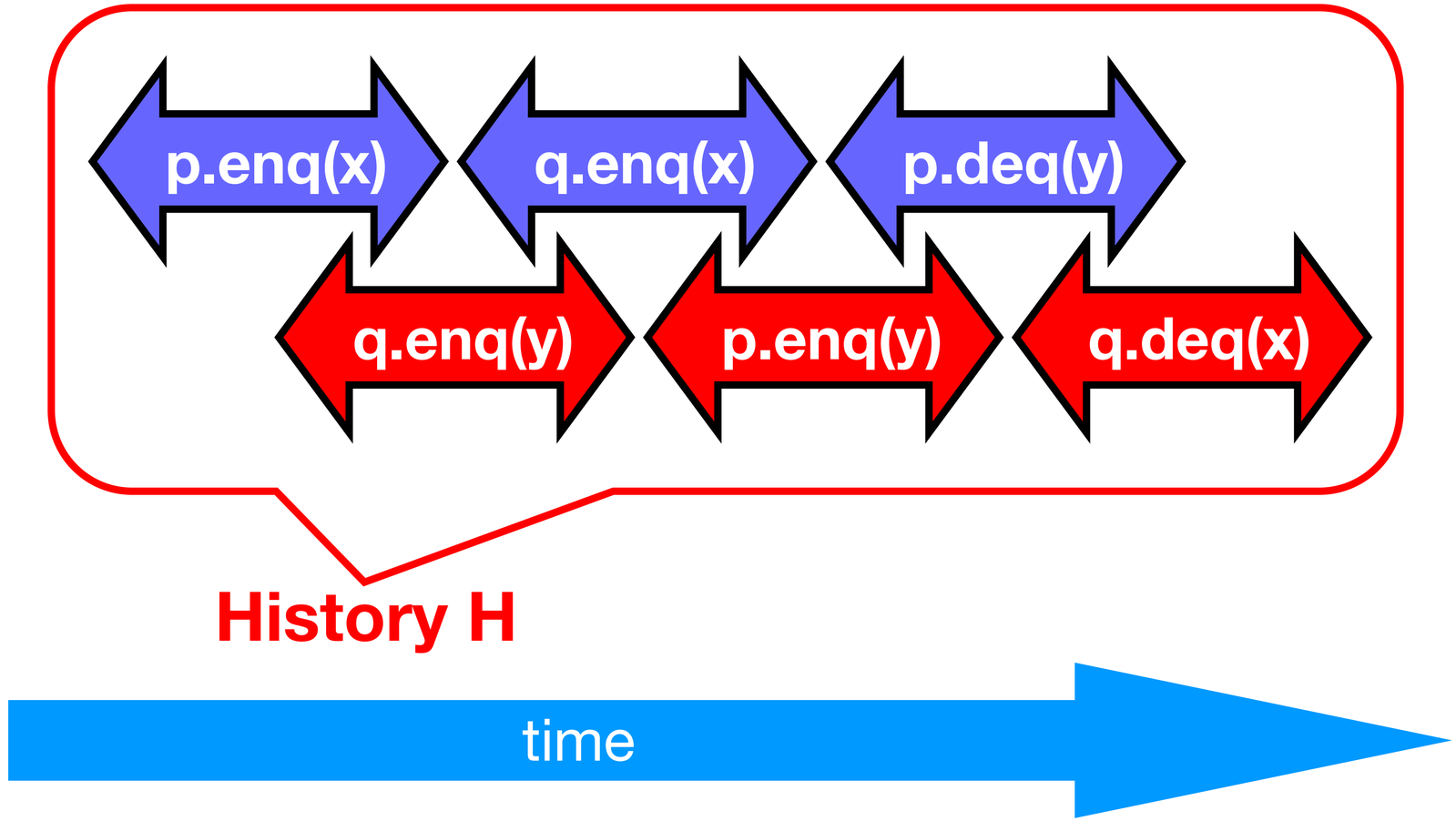
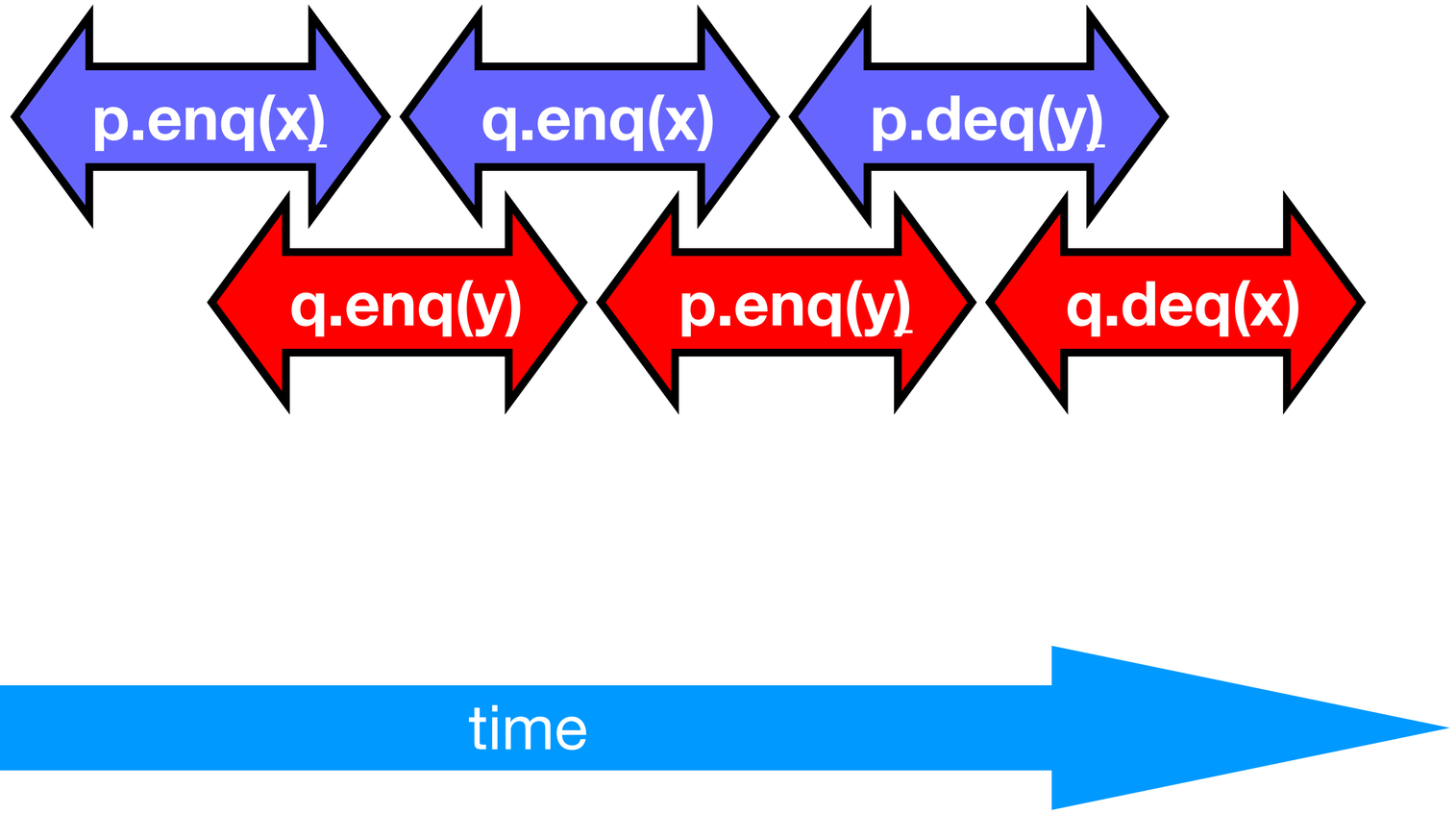
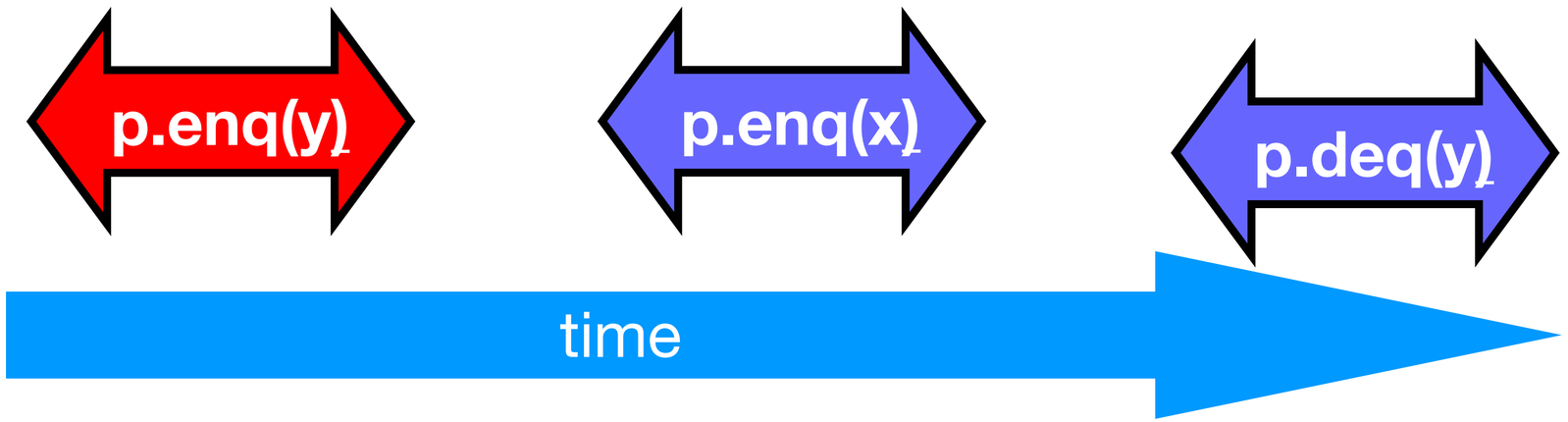


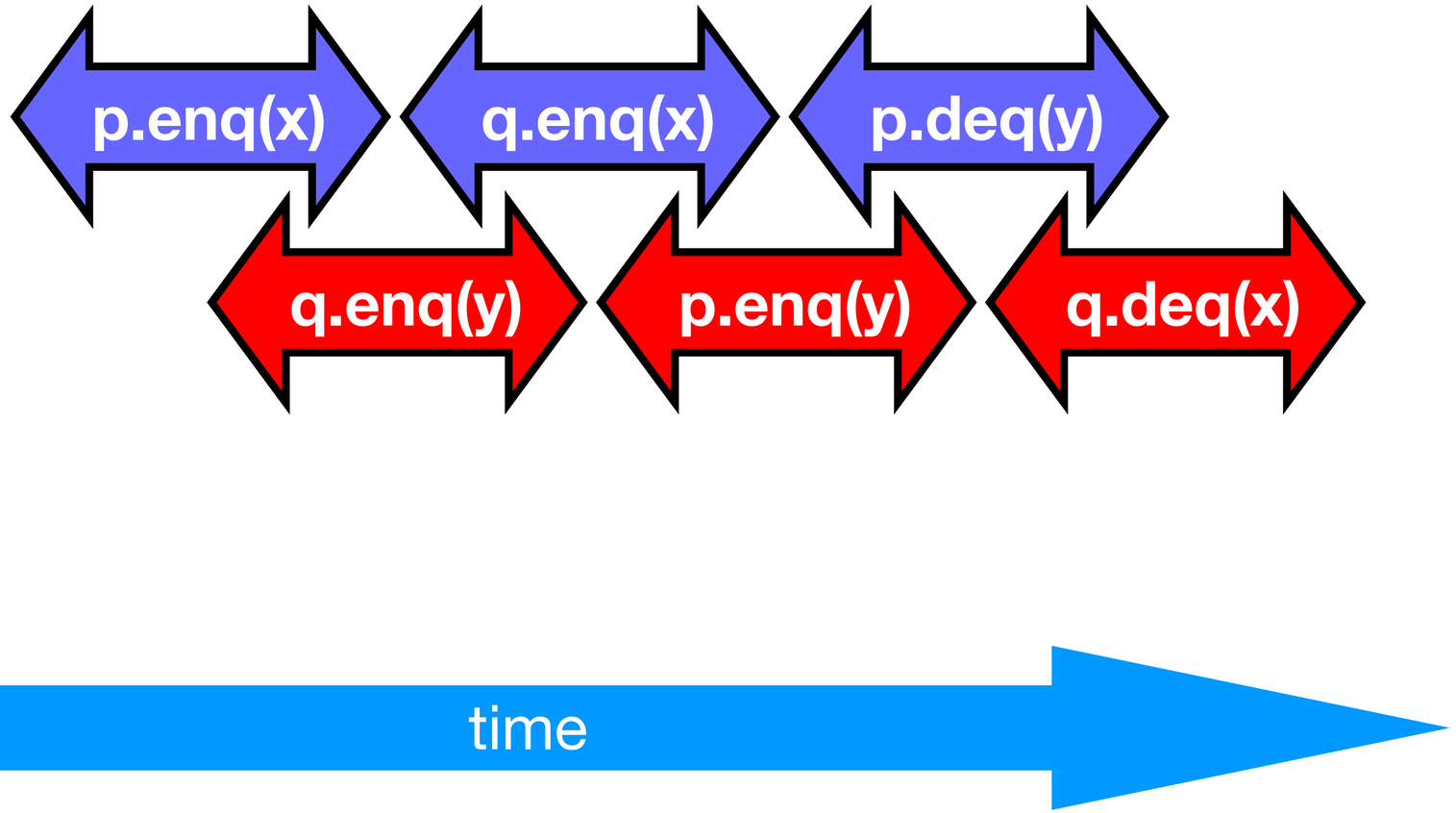
H|p Sequentially Consistent



H|p Sequentially Consistent



H|q Sequentially Consistent



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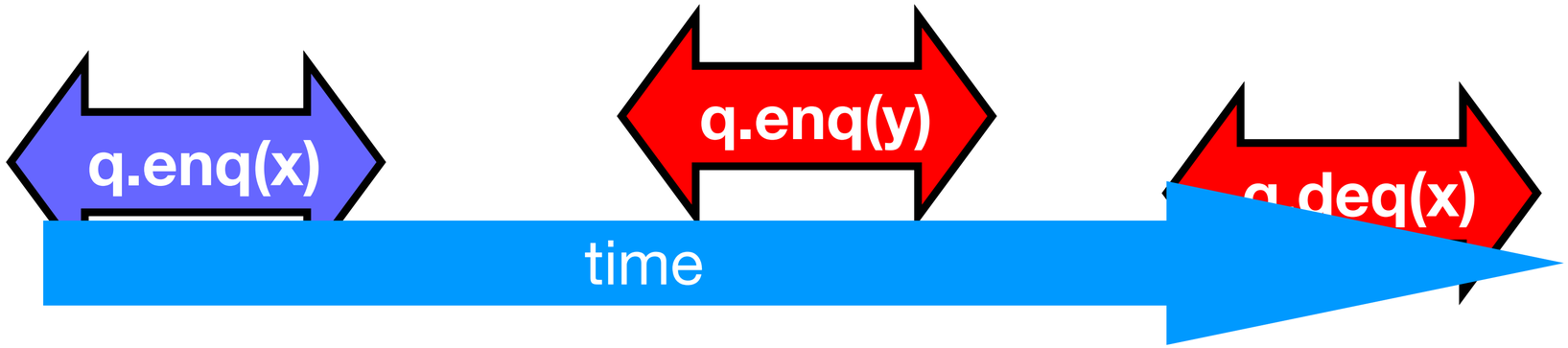
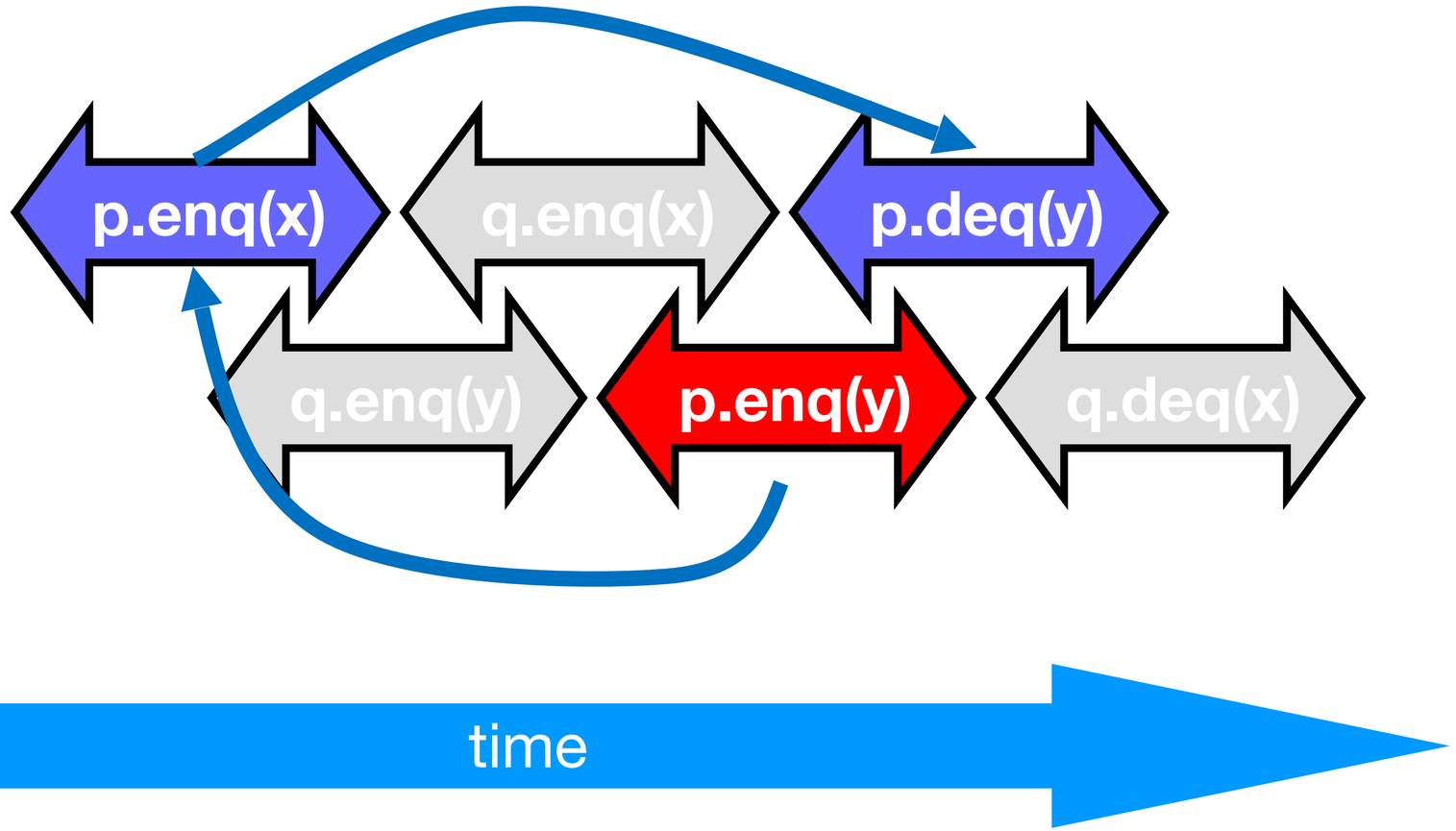
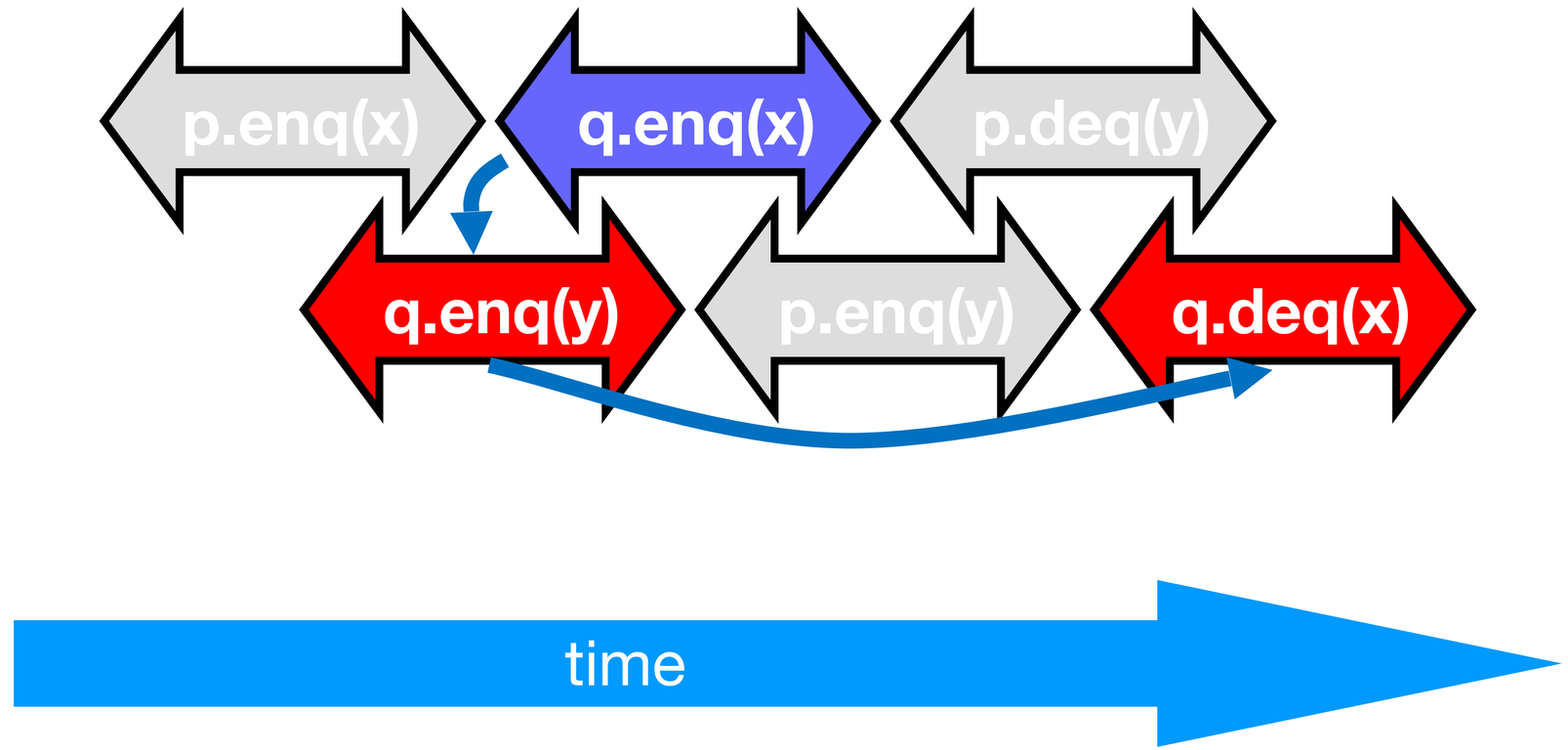


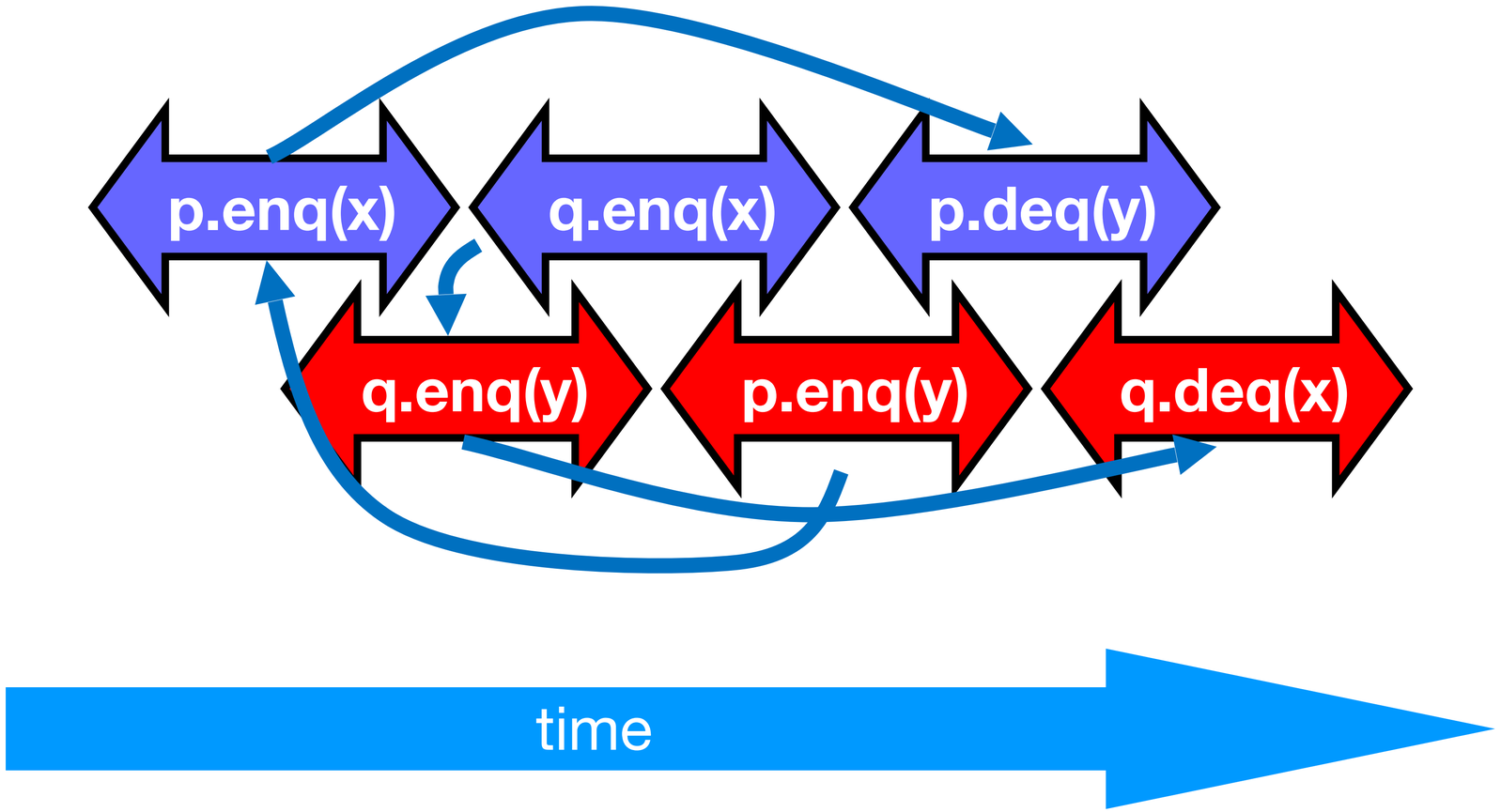
Ordering imposed by p



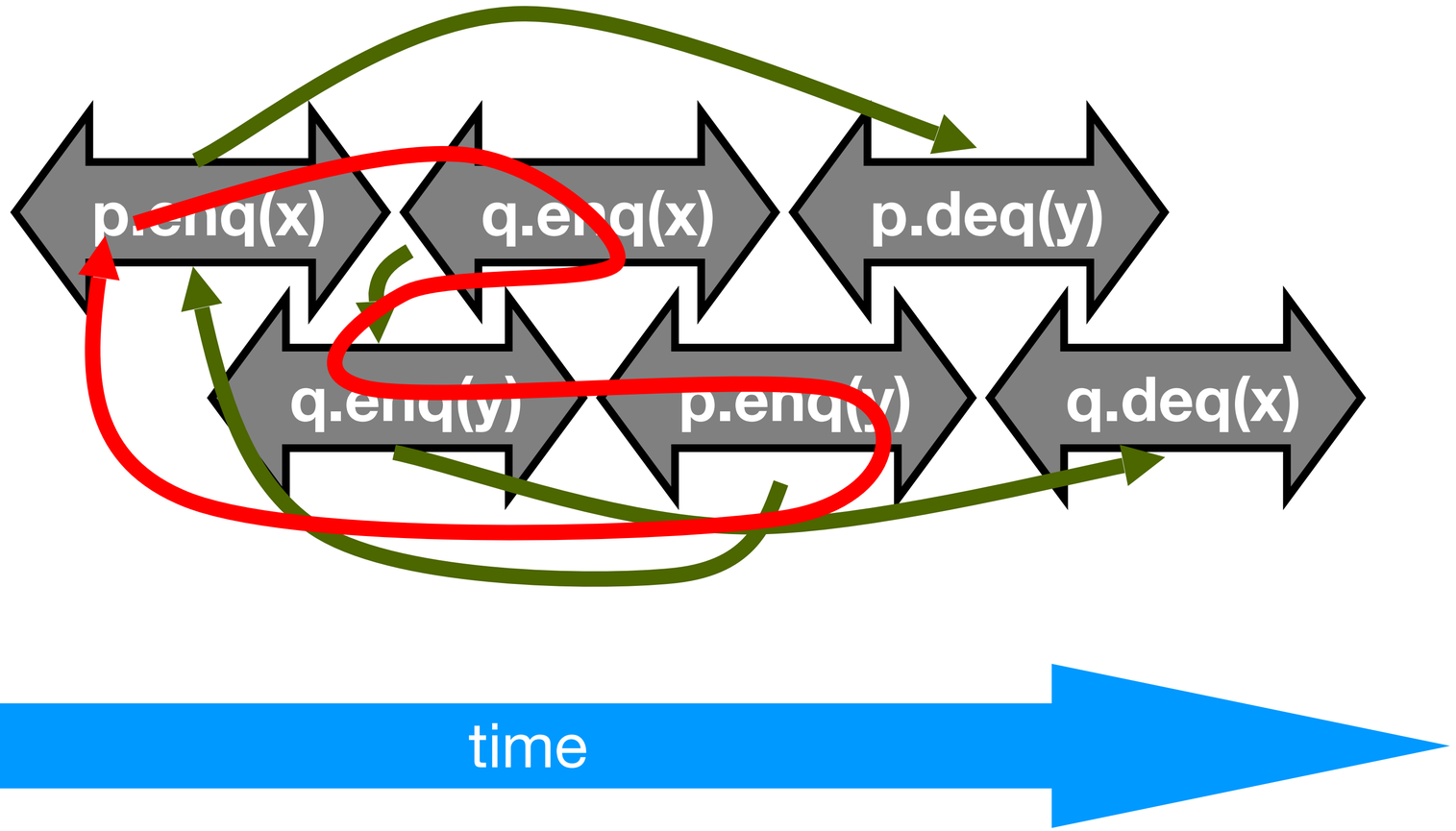
Ordering imposed by q



Ordering imposed by both



Combining orders



Concurrency Testing

- Linearizability and Sequential Consistency are good specifications for *testing* the correctness of concurrent data structures.
  - Any observed execution must match a sequential execution
  - Can be exploited for pragmatic testing
- See https://github.com/ocaml-multicore/multicoretests

qcheck-lin

- Checks for sequential consistency violations (despite what the name says)
- ***Every sequential consistency violation is a linearizability violation***
- Check that the observed result of a parallel implementation can be observed with a sequential run

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qcheck-stm

- In ***qcheck-lin,*** what if the implementation is wrong for the sequential program itself?
  - We're only comparing equivalence.
  - Sequential run of a buggy implementation ≡ Parallel run of a buggy implementation
    - ***Is not useful!***

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Summary

- ***Linearizability***
  - The operation takes effect instantaneously between the invocation and the response
  - Uses sequential specification, locality implies composability
- ***Sequential Consistency***
  - Linearizability without real-time ordering
  - Not composable
  - Harder to work with
  - Useful to reason about hardware models (next lecture)
- We will use ***linearizability*** as our consistency condition for reasoning about objects

Progress

- We saw an implementation whose methods were lock-based (deadlock-free)
- We saw an implementation whose methods did not use locks (lock-free)
- How do they relate?

Progress Conditions

- ***Deadlock-free:*** some thread trying to acquire the lock eventually succeeds.
- ***Starvation-free:*** every thread trying to acquire the lock eventually succeeds.
- ***Lock-free:*** some thread calling a method eventually returns.
- ***Wait-free:*** every thread calling a method eventually returns.

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We will look at linearizable blocking and non-blocking implementations of objects.